Signaling Valence in Primary Elections*

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April 2, 2018

Abstract

I build a model of two-stage elections in which candidates differ in terms of a privately observed quality dimension (valence) and they commit to a policy platform before the primary election. Primaries select better candidates at the cost of increased polarization of platforms, resulting in a welfare loss for the median voter. I also endogenize the choice of holding primaries and find that it can lead to asymmetric equilibria, in which only one political party holds primaries.

*This project is part my PhD thesis. I am very grateful to my supervisors Andrea Mattozzi and David Levine; I would also like to thank Johanna Reuter and Christopher Li, as well as audiences at the European University Institute and the 2016 EEA-ESEM Meeting in Geneva, for stimulating questions and comments.

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1 Introduction

"*Mr. Trump's outrageous statements signal that he has some other political virtue some voters value.*"

Justin Wolfers, New York Times, 2015

Many of the characteristics that make a politician successful cannot be captured by policy platforms. Integrity, honesty, like-mindedness are examples of the qualities voters look for in a politician; although these traits can play a key role in deciding the outcome of an election, they are difficult to credibly communicate. This problem is particularly salient when trust in political institutions is low.[1]

This paper develops a simple model in which office-motivated politicians use extreme electoral platforms to signal their quality, which I denote as valence. The key institutional feature that allows politicians to credibly signal their valence is a primary election.

The model has a number of interesting implications: first of all, it suggests that, under some circumstances, ideologically extreme policy platforms are a result of the effort by candidates to signal their valence. An intriguing example in this respect is Donald Trump's primary election campaign: his markedly extreme proposals differentiated him from other candidates, allowing him to gain the trust of large swathes of voters as an anti-establishment candidate. As Wolfers (2015) writes in the New York Times, "Trump's outrageous statements signal that he has some other political virtue some voters value" and his statements might be "less a calculated attempt at feeding a demand for bigotry and more an effort to fuel the hunger for authenticity".

In this respect, my model predicts that an increase in policy polarization can occur due to valence becoming less observable, absent any changes in the ideology of party voters; for example, valence may become less observable because of changes in the qualities desired in a politician or following a decrease in the level of trust in political institutions. This result speaks to existing evidence on the fact that the recent rise in polarization in the US has not been paralleled by increased polarization in party voters, as argued for example by Fiorina et al. (2006).

From the point of view of institutional design, my analysis suggests that when primaries are used by candidates to signal valence, they increase the quality of candidates but they also contribute to policy polarization, generating a trade-off for social welfare.

[1] A poll providing a good example of what are the characteristics voters find important in a candidate can be found here: http://news.gallup.com/poll/12544/values-seen-most-important-characteristic-presidential-candidates.aspx

The fact that primaries tend to select better candidates is supported, among others, by Snyder and Hirano (2014), who show that the effect is particularly relevant in districts which are electorally safe for the party holding them. Concerning the effect of primaries on polarization, Hirano et al. (2010) and McGhee et al. (2014) find little or no effect, whereas Kaufmann et al. (2003) and Gerber and Morton (1998) suggest that closed primaries select more polarized candidates than open primaries, and Bullock and Clinton (2011) find a polarizing effect of primaries only in competitive districts. Regarding the correlation between extremism and valence, the evidence is mixed: on one hand there is some evidence that valent primary candidates choose more moderate policies[2], but there are also elements pointing in the opposite direction: for example, Stone and Simas (2010) shows that challengers choosing more extreme platforms have a higher probability of replacing the incumbent in House elections; similarly, Brady et al. (2007) and Hirano et al. (2010) show that ideologically moderate incumbents are more vulnerable to primary challenges. In this respect, the aim of my paper is not to argue that one should expect an unconditional positive correlation between valence and extremism, but to suggest that in some circumstances - for example primary races with relatively unknown challengers - one can see a positive correlation.

Finally, I also show that if parties are free to choose whether to hold primaries, their decision will sometimes be asymmetric, that is only one party will hold primaries in equilibrium: this matches the observation that, especially in European and South American countries, only some parties organize primaries.[3]

The model I develop considers a two-stage election. Two candidates are drawn to compete in a party primary, the winner running as the party candidate in a general election against an incumbent from the opposing party. Candidates are purely office-motivated, but they commit to a policy platform when entering the primary election race.[4] The key feature of the model is that valence is private information of candidates at the time of the primary election, but the electoral campaign following the primaries can expose a candidate as non-valent. In this environment, the primary election gives valent candidates the opportunity to credibly signal their valence by committing to a more extreme policy platform compared to their non-valent opponents. The reason is

[2]See for example Ansolabehere et al. (2001), Stone and Simas (2010) and Brady et al. (2007)

[3]See Carey and Polga-Hecimovic (2006) and Kemahlioglu and Hirano (2009) for a discussion of the use of primaries in Latin America; in Europe, primaries have been used in several countries including Italy, France, Greece and Spain.

[4]The assumption of commitment to policy platforms is common to a large number of papers on primary elections, and is also supported by empirical evidence, see for example Poole and Rosenthal (1997). For a discussion of this and other of my modeling assumptions, please refer to Section 3 and Section 4.1.

that competing in the general election with an extreme platform is more costly for a non-valent candidate, since the campaign would reveal their lack of valence and jeopardize their chances of electoral victory. This allows for a separating equilibrium in which valent candidates choose more extreme platforms: hence, primaries select better candidates, but they do it at the cost of increased polarization.

Another relevant insight provided by my paper is that the order in which platform commitment and valence revelation occur is key to determine the outcome of a primary election competition. If valence is revealed before politicians commit to a policy platform, such as in Hummel (2013), valent politicians acquire a bargaining power that allows them to offer more moderate policy platforms than those of their non-valent opponents. If however valence is not observable when politicians choose their platforms, valent politicians have to engage in costly signaling by choosing more extreme platforms. The feature of privately observed valence makes this model particularly apt at describing the primary election strategy of dark-horse candidates, or primary elections at times of low trust in party institutions, during which it is more likely for candidates to have the necessity of convincing voters of their valence.

The paper is structured as follows: Section 2 reviews the related literature, Section 3 presents the model in which only one party holds a primary election: I call this the single-primary model. The main results from this model are described in Section 4, along with a discussion and welfare analysis. After that, Section 5 presents a more general version of the model, in which both parties hold primaries. The case of endogenous primaries is also considered there. Section 6 concludes.

2 Related Literature

A paper closely related to mine is the work by Kartik and McAfee (2007). In their model, candidates with character are unstrategic and choose a policy according to some underlying distribution. Strategic candidates, on the other hand, locate using an optimal randomization strategy which follows the distribution of candidates with character but is biased towards the centre. The relation between character/valence and relatively extreme positions is thus a common prediction of both models. In my setup, however, all candidates are strategic and it is up to valent candidates to separate from their opponents. Moreover, unlike in Kartik and McAfee (2007), my analysis isolates an institutional element, that is primaries, as the key factor that allows valent candidates to separate.

Another important reference is Hummel (2013): the two models share a similar spatial setup, with one crucial difference. As anticipated in the introduction, I assume valence is privately observed at the time of primary elections and only revealed during the electoral campaign, when politicians have already committed to their policy platforms, whereas in the model by Hummel (2013) valence is publicly observable before the primary election. This simple difference gives rise to the signalling dimension, which is the main contribution of my work.[5]

The question of when valence is revealed and how that affects primaries is at the core of the study by Snyder and Ting (2011); in their model, the type of politician can be revealed either in the primary or in the general election campaign. Therefore, the cost of primaries is that they increase the probability that a bad a candidate is identified before the general election, damaging the party; in my model, it is precisely the fact that the electoral campaign can reveal a bad candidate which makes primary elections useful. The crucial difference between the two models is that in their model there can be no signaling of valence, since candidates are not informed about their valence, and the policy positions of each party are fixed.

A model of primary elections which also shares some insights with mine is Casas (2014). His paper focuses on the difference between open and closed primaries in a setup where policy preferences are not observable; a similar insight emerging from his analysis is that valent candidates are more likely to be extreme, in his case because of more volatile policies.

The trade-offs related to holding primaries are also considered in the paper by Serra (2011), who models a situation in which primaries increase the probability of choosing a valent candidate but reduce the party control over policy platforms. However, whereas the increased probability of valence is an assumption of his model, the signaling logic I develop allows me to describe increased valence as an equilibrium outcome. Another paper that investigates the institutions governing party organization is Crutzen et al. (2009): in their model, primaries provide an incentive to candidates to exert effort, and when information on platforms is low this leads to platforms of higher quality. In Crutzen et al. (2009) it is therefore the choice of holding primary elections and not the choice of platforms that works as a signal.

A study which does not consider primaries but that also offers an explanation of strategic extremism is Eguia and Giovannoni (2017): in their model, disadvantaged candidates choose an extreme platform which allows them to build credibility as alternative candidates for future elections. A key difference lies in the fact that in their setup valence

[5]Elections where candidates can have a valence advantage are also studied in Aragones and Palfrey (2002) and Groseclose (2001).

is defined as policy-specific competence, which a party can acquire by owning the issue: in my model, on the other hand, valence is candidate-specific and orthogonal to policy.[6]

3 The Model

I construct a two-stage election game based on a standard Downsian spatial voting model. Voters have preferences distributed over the $x \in [0, 1]$ interval. The location of the median voter, denoted by m, is a random draw from a uniform distribution in the $[\frac{1}{2} - \epsilon, \frac{1}{2} + \epsilon]$ interval. There are two political parties, whose policy bliss points are fixed at the extremes of the interval: I call the party located at $x = 0$ Democratic party and the party located at $x = 1$ Republican party, or D and R respectively. The policies proposed by each party in the general election are denoted by r and d respectively. The voters of each party are homogeneous and the size of the two parties is identical: therefore, we can think of a representative voter in each party. Party affiliation is fixed.

In the baseline version of the model I assume that the Republican candidate for the general election is an incumbent located at r, whereas the Democratic party holds primaries to establish who is going to be its nominee. Two candidates are randomly drawn from an exogenously given pool of politicians to run in the Democratic primary. As soon as they are drawn, each of them proposes a policy platform: the policy platform cannot be changed and it gets implemented if the candidate wins the general election. Having observed the platforms chosen by the candidates, the Democratic party representative voter chooses one of them to run in the general election. The primary election I model is therefore a closed primary, since only party voters participate. The winner of the Democratic primary becomes the opponent of the Republican incumbent. The location of the median voter is then realized and determines the winner of the general election. Finally, the winning policy is implemented, payoffs are distributed and the game ends.

Politicians are purely office motivated, receiving a utility of 1 if elected in the general

[6] There are a number of other models of primary elections: Owen and Grofman (2006) study a two-stage spatial model of elections; Grofman et al. (2016) consider both closed and open primaries with candidates of potentially different observable valence and show that primaries benefit the party closer to the general population median; Takayama (2014) presents a model with observable valence similar to Hummel (2013) in which primaries lead to policy moderation; Serra (2015) shows that policies can converge to the median despite the presence of primaries; Adams and Merrill (2008) build a model where myopic voters in a primary pick better candidates located between the party and general election median voter; Meirowitz (2005) presents a model in which primaries allow candidates to learn the preferences of voters; Buisseret and Van Weelden (2017) study the trade-off for a non-establishment candidate between entering the party primary or competing as a third candidate. Finally, Hummel (2010) and Agranov (2016) focus on the flip-flopping of candidates between primary elections and the general election, which is not part of my model, since politicians commit to their platform.

election, independently of the policy chosen, and 0 otherwise.[7] There are two types of politicians: valent and non-valent, or H and L. The share of valent politicians in the candidate pool is fixed at α. Valence represents those characteristics of a politician that are orthogonal to the proposed policy platform and that provide utility to all voters across the ideological spectrum.[8] I assume that valence, denoted by ν, is binary, so $\nu \in \{0, v\}$, where $v > 0$ denotes the valence level of valent politicians. The valence of the Republican incumbent is fixed at 0, such that valence can also be interpreted as valence advantage over the incumbent.

The utility a voter located at x derives from politician j winning the general election consists of the sum of the politician's valence and the distance between the candidate's policy platform p_j and the voter's bliss point x:

$$u(p_j, \nu_j, x) = \nu_j - |p_j - x|$$

When the primaries take place, valence is private information of politicians. I assume that after primaries, the valence of general election candidates is exogenously revealed, for example as a result of media scrutiny.[9] When voting in the primary stage, party members are forward looking: they choose the candidate maximizing their expected utility, taking into account the candidates' policy platforms, valence and the probability of each candidate to win the general election.

A convenient piece of notation that I will use throughout the paper is $\tilde{r} \equiv r - (\frac{1}{2} - \epsilon)$, which denotes the distance between the Republican policy and the left extreme of the median voter distribution. Similarly, $\tilde{d} \equiv \frac{1}{2} - \epsilon - d$. I further assume that valence is not too large, and in particular that $v \leq \tilde{r}$. This assumption makes sure that a valent Democratic candidate located at $\frac{1}{2} - \epsilon$ would not win the election for sure. In the text I will denote by $\tilde{\rho} \equiv \frac{v}{\tilde{r}}$, which will be useful to express some of the results.

Assumption 1. *$v \leq \tilde{r}$, or $\tilde{\rho} \leq 1$*

Before delving into the analysis of the model, I sum up the timing of the game:

[7] My results would not change if I assumed that winning the primary gives a politician utility independently of the general election victory.

[8] These characteristics can be thought of as honesty, work ethics, international reputation but also as campaigning ability. No result other than the welfare of the Republican voter changes if we instead assume that only party members and the median voter derive utility from valence.

[9] Another reason for valence to be revealed after the primary election is if one thinks of it as general election campaigning ability, that is how well the candidate's message will resonate with the median voter. The assumption of private information is particularly relevant if valence is correlated with backing from donors who are waiting for the general election to release their financial firepower. In an extension, see Appendix C, I consider the case in which valence is not always revealed before the general election.

1. Two Democratic primary candidates are drawn.
2. Primary candidates simultaneously choose their policy platforms.
3. The Democratic voter observes the platforms and chooses one candidate.
4. The valence of the democratic candidate is observed.
5. The general election takes place.
6. The winning policy is implemented and payoffs are distributed.

4 Results

I solve the game backwards, starting from the general election.

After the primary has determined the Democratic candidate, whose platform I denote by d, and valence is revealed, all players vote for their favorite candidate. The winning party is determined by the location of the median voter m compared to that of the hypothetical voter indifferent between the two candidates, denoted by $z \equiv \frac{d+r+\nu}{2}$: D wins if $m \leq z$ and R wins if $m > z$. The probability of winning for the Democratic candidate is therefore:

$$\pi(d, r, \nu) = \frac{1}{2} + \frac{1}{4\epsilon}(d + r + \nu - 1)$$

The value of π is important since it represents the politicians' payoff conditional on winning the primary with policy d.

Before discussing the primary election it is useful to consider as a benchmark the platform that, everything else equal, maximizes the expected utility of the Democratic party voter; for a primary candidate located at d and with valence level ν this can be denoted by:

$$\begin{aligned} U_D(d, \nu, r) &= \pi(d, r, \nu)u(d, \nu, 0) + (1 - \pi(d, r, \nu))u(r, 0, 0) \\ &= \pi(d, r, \nu)(\nu - d) - (1 - \pi(d, r, \nu))r. \end{aligned}$$

When picking a candidate from the pool, the expected utility of the Democratic party voter is $U_D(d, v, r)$ with probability α and $U_D(d, 0, r)$ with probability $1 - \alpha$. Maximizing this expected utility with respect to d turns out to be particularly simple thanks to the linearity of the utility function: neither valence nor the opponent's location affect the utility maximizing location of the Democratic candidate, which uniquely depends on the median voter location uncertainty parameter ϵ:[10]

Lemma 1. *Everything else equal, the candidate location maximizing the utility of the Democratic party is* $d^* = \frac{1}{2} - \epsilon$.

Proof. See Appendix. □

I can now move on to the analysis of the primary election, which is the heart of the model. Before the primary elections, candidates pick a policy platform $p(\nu) \in \mathbb{R}$ based on their type. I focus on pure strategies, so $p(\nu)$ is simply a real number. Having observed the platforms p_j and p_{-j} of the two primary candidates, the Democratic voter updates her beliefs on the valence of each politician to $\nu(p)$ and chooses the candidate providing her with the highest expected utility $U_D(p_j, \nu(p_j), r)$. In a separating equilibrium, $p(v) = d_H$ and $p(0) = d_L$, with $d_H \neq d_L$, and the Democratic voter picks a valent candidate whenever one is available: if two identical candidates are available, the voter chooses each of them with probability $\frac{1}{2}$. As a result, the probability for a valent candidate to win the primary is $w_H = 1 - \frac{\alpha}{2}$; a non-valent candidate, instead, wins the primary with probability $w_L = \frac{1-\alpha}{2}$. Clearly, $w_H > w_L$. As a result, the expected utility of a valent primary candidate in a separating equilibrium is $U_c(d_H, v) = w_H \pi(d_H, v, r)$, and that of a non-valent candidate is $U_c(d_L, 0) = w_L \pi(d_L, 0, r)$. In order for a separating equilibrium to be sustainable, two conditions have to be satisfied. The first is incentive compatibility, that is requiring that non-valent politicians have no incentive to mimic their valent opponents:

$$U_c(d_L, 0) \geq U_c(d_H, 0) \tag{1}$$

This condition summarizes the signaling logic underlying the result: a non-valent politician does not try to mimic the valent one when his lower probability of winning the primaries (captured by $w_L < w_H$) is compensated by the higher probability of winning the general election, conditional on winning the primary. The latter is higher when the non-valent politician sticks to the non-valent policy, since valence is revealed by the electoral campaign and $\pi(d_H, 0, r) < \pi(d_L, 0, r)$.

The second condition for a separating equilibrium requires that Democratic voters are willing to vote for the valent candidate at d_H rather than for the non-valent candidate at d_L:

$$U_D(d_H, v, r) \geq U_D(d_L, 0, r) \tag{2}$$

[10] This simple benchmark is ideal in order to isolate the effects of signaling in the primary election. Nevertheless, the separating logic I describe would remain qualitatively unchanged under a different benchmark.

The reason why this condition arises is that in this model the signaling effort is not only costly for the agent (the politician) but also for the principal (the party voter); remember that by Lemma 1, under full information the party voter would prefer the politician to locate at d^*. If the signaling policy d_H is too extreme, therefore, signaling is not feasible. When a separating equilibrium does not exist there can be a pooling equilibrium, but it can also be that no pure-strategy equilibrium exists in that case.[11]

The following condition describes the parameter restriction necessary in order for (2) to be satisfied conditional on (1) holding.[12] A separating equilibrium is sustainable when the ratio $\tilde{\rho}$ between valence v and the extremism of the Republican incumbent $\tilde{r}$ is larger than the threshold $\tilde{\rho}_1(\alpha)$, which depends on α. In other words, valence has to be large enough and the Republican incumbent has to be located sufficiently to the right of the political spectrum.

Condition 1. $\tilde{\rho} \geq \tilde{\rho}_1(\alpha)$

Derivation. See Appendix. □

I am now ready to summarize the main result of the paper in the following theorem:

Theorem 1. *Assume that $\tilde{\rho} \in [\tilde{\rho}_1(\alpha), 1]$, so that Assumption 1 and Condition 1 hold. Then, the Democratic primary election game has a pure-strategy Perfect Bayesian Equilibrium (PBE) in which low-valence candidates locate at $d_L = d^* = \frac{1}{2} - \epsilon$ and high-valence candidates locate at $d_H = \frac{1}{2} - \epsilon - \frac{1}{2-\alpha}\tilde{r}$. This equilibrium is the only one surviving the Intuitive Criterion.*[13]

Proof. See Appendix. □

This theorem states that valent politicians signal their type by choosing a more extreme policy than their non-valent opponents. The credibility of the signal relies on the fact that non-valent politicians find it costly to run on an extreme platform, since they have no valence to make up for the distance from the median voter.

Valent democratic politicians send a credible signal by moving to the left of $\tilde{d}_H = \frac{1}{2-\alpha}\tilde{r}$. This means that the higher α, the more radical the signaling policy becomes: when α moves closer to 1, the separating policy gets closer to the policy with zero winning probability in the general election for a non-valent candidate mimicking the valent type's

[11] A pooling equilibrium does not exist when a valent politician finds it profitable to separate given that other valent politicians keep pooling. More details can be found in the Appendix.

[12] As I explain in the appendix, the condition is made more restrictive in order to also make sure that no pooling equilibrium survives the Intuitive Criterion.

[13] SeeCho and Kreps (1987).

policy. This follows from the fact that as α goes to 1, the probability of winning the primary election for a non-valent politician goes to zero. Moreover, notice that $\tilde{d}_H$ remains bounded away from zero even when α is very close to zero. The reason is that independently of how small α is, locating at d_H has a large effect on the probability of winning the primary election, and therefore if d_H was very close to d^*, there would be a discontinuous jump in $w(p)$ moving only a little bit to the left of d^*, which cannot be incentive compatible.

The other comparative statics consider parameter changes that affect the probability of the low type to win the general election. A decrease in this probability requires an increase in d_H (that is a more moderate signaling policy) and the other way around. For example, the further to the right the location of the Republican opponent is, the more movement to the left is required from the Democratic candidate. In other words, primaries contribute to political polarization. Notice that $\tilde{r}$ can also be interpreted as a parameter representing district safety or the advantage of the Democratic party. Another possible interpretation is therefore that primary elections make valent candidates more extreme in safer districts. Notice that an increase in $\tilde{r}$ can be due to either an increase in r or an increase in ϵ: hence, also greater uncertainty on the location of the median voter leads to greater polarization of the separating platform d_H.

The valence of the Democratic politician does not affect the value of d_H, since this depends on the probability that the non-valent candidate wins the general election: even following a deviation to d_H, this probability does not depend on valence, given the assumption of exogenous valence revelation before the general election. Finally, if I allowed the Republican candidate to have a valence level different from 0, d_H would increase, that is the signaling location would become more moderate, under the same logic following a movement of r to the left.[14]

These findings can be summarized in the following corollary:

Corollary 1. *The policy platform of the valent candidate d_H moves to the left as α or $\tilde{r}$ increase, the latter meaning that either r or ϵ increase.*

In order to make other comparisons, we can consider as no-primary benchmark a situation in which the party picks a politician from the pool of candidates and the nominee runs with platform d^*. This is observationally equivalent to a pooling equilibrium at d^*. With this benchmark at hand, a number of conclusions can be drawn. First, in terms of

[14] Adjusting the model to deal with a valent opponent is straightforward and yields a separating location of $\tilde{d}_H = \frac{1}{2-\alpha}(\tilde{r} - v)$.

total probability of winning the general election, a unilateral primary is always damaging for the Democratic party, despite the increased probability of having a valent candidate:

Corollary 2. *A unilateral primary never increases the probability of the Democratic party to win the general election.*

This result is driven by the fact that a unilateral primary skews the policy of the Democratic party to the left without changing the platform of the Republican party. As I will show in Section 5, this ceases to be the case when both parties hold primaries.

Conditional on winning the primary election, valent candidates win the general election more often than non-valent ones as long as $d^* - d_H \leq v$. This requires that $\tilde{\rho} \geq \frac{1}{2-\alpha}$. In terms of unconditional probability of winning office for a Democratic candidate, this increases as long as $\tilde{\rho} \geq \frac{\alpha}{1-\alpha}$. Given the restriction that $\tilde{\rho} \leq 1$, this means that a necessary condition for the unconditional probability of seeing a valent candidate in office to increase is $\alpha \leq \frac{1}{2}$. Another interpretation of this result is that primaries increase the utility of valent candidates only if the pool is sufficiently bad.

Corollary 3. *Conditional on winning the primaries, valent Democratic candidates win the general election more often than non-valent ones if and only if $\tilde{\rho} \geq \frac{1}{2-\alpha}$. Unconditionally, valent Democratic candidates win more often if and only if*

$$\tilde{\rho} \geq \frac{\alpha}{1-\alpha}. \tag{3}$$

To sum up, primaries are beneficial to valent candidates as long as i) valence is sufficiently high, ii) the Republican opponent is sufficiently moderate (or the district is sufficiently competitive), iii) uncertainty on the location of the median voter is sufficiently high or iv) the pool of politicians is sufficiently bad. These observations can be useful in guiding the interpretation of empirical correlations between valence, ideological location and the probability of winning office.

The results on winning probabilities naturally lead to the discussion on the welfare effect of primaries on the remaining players of the model: since a unilateral primary always decreases the winning probability of the Democratic party, the welfare of the Republican incumbent increases. For the same reason, the welfare of a Democratic candidate not yet aware of his type also decreases. This means that under a veil of ignorance concerning their valence, Democratic politicians would never be in favor of holding a unilateral primary. On the other hand, the welfare of valent Democratic candidates can increase. It does so if and only if their unconditional probability of winning the election increases, that is if and only if $\tilde{\rho} \geq \frac{\alpha}{1-\alpha}$. Finally, the effect of primaries on non-valent candidates is always negative, since all what changes for them is the probability of winning the primary election (conditional on being drawn), which decreases from $\frac{1}{2}$ to $\frac{1-\alpha}{2}$.

The effect of primaries on party voters is more nuanced, since they also value the implemented policy. The Democratic party trades off some probability of winning the general election with more valence and an average policy platform, conditional on winning, closer to their bliss point. It turns out that the welfare of the Democratic voter increases if and only if $\tilde{\rho}$ is larger than a threshold denoted by $\tilde{\rho}_2(\alpha)$ (details in the Appendix). As α grows closer to one, $\tilde{\rho}_2(\alpha)$ grows unboundedly; this means that there exists an upper bound on α past which the party never gains from primaries (under $\tilde{\rho} \leq 1$). This is intuitive since when it is very likely to pick a valent politician from the pool, there is little reason to hold a primary.

Corollary 4. *Primaries benefit the Democratic party voter if and only if $\tilde{\rho} \geq \tilde{\rho}_2(\alpha)$.*

One of the consequences of these welfare effects is that the interests of party voters and valent candidates need not be aligned. From (3) and the expression of $\tilde{\rho}_2(\alpha)$ one derives that for high values of α, $\tilde{\rho}_2(\alpha) > \frac{\alpha}{1-\alpha}$. This means that if α is large enough, it is possible for parties to gain from primaries when politicians don't, and the other way around if α is low enough.

Finally, it is worth considering the effects of primaries on the median voter. In order to do that, I proceed by verifying for each possible realization of the median voter m whether she would benefit from the adoption of primaries in the Democratic party. The space of possible realizations of the median voter can be divided into several portions according to the different types of voting behavior, ranging from the median voter types who always vote for r independently of the adoption of primaries, to median voters who always vote for d. It turns out that whereas the median voter types on the extreme right of the spectrum are indifferent to the adoption of primaries, under Assumption 1 all other median voter types are made strictly worse off by the Democratic party primary. This result is related to Corollary 2, that is the fact that primaries decrease the probability for the Democratic party to win the general election.

Corollary 5. *For all realizations of her location m, the median voter is either indifferent or worse off when the Democratic party holds a primary.*

In the next paragraph I discuss some of the model assumptions and further elaborate on how my results relate to some of the empirical literature on primaries and candidate valence.

4.1 Discussion

The main insight of this paper is to show that strategic politicians can use extreme policy platforms in order to signal their valence to voters in a primary election. The key point where my model departs from the existing literature on primaries is the inversion of the order in which policy platforms are set and valence is revealed. Whereas the usual assumption is that valence is public information when politicians choose their platform, here I study what happens when politicians have to choose their platforms before valence becomes public information. Of course, this need not be the ideal specification to study all primary elections: however, there are situations where it is reasonable to believe that a politician's valence is not known at the time of primary elections. This can be the case when a candidate is not well known at the time of the primary race, or when we interpret valence as representing qualities correlated with the ability in campaigning for the general election. As a concrete example, suppose valence is the absence of scandals that can be damaging in the general election. With media scrutiny concentrating on presidential candidates before the general election, a scandal which has not become public before primaries is likely to do so before the general election. In the baseline model I assume that valence is always exogenously revealed before the general election, but in Appendix C I consider an extension in which that happens with probability lower than 1, as in the aforementioned paper by Snyder and Ting (2011).

The second important assumption for my results is commitment to policy platforms. This assumption is very common in the literature on elections, including primary elections.[15] Moreover, there is empirical evidence, see for example Poole and Rosenthal (1997), that politicians commit to their campaign promises; there is also evidence that ideological re-adjustment is costly and voters usually punish politicians flip-flopping on a policy stance.[16] In my model, commitment ensures that non-valent politicians attempting to mimic valent ones bear the electoral cost of an extreme policy, setting the incentives for a separating equilibrium. However, this assumption can be relaxed without affecting the main insight of the model: as I show in Appendix D, if I assume that politicians can relocate after the primary with some probability $k > 0$, the incentive compatibility constraint can be rewritten in a very similar fashion, which reflects the increased cost of signaling due to the possibility of relocating.

Finally, another important element for some of the results is the no-primaries benchmark. The way I model it is by assuming that when primaries are not held, the party

[15] This assumption is made by Kartik and McAfee (2007), Hummel (2013), Grofman et al. (2016), Takayama (2014) among others.

[16] See for example Tavits (2007) and DeBacker (2015).

chooses a platform and picks a random candidate from the pool. This makes the no-primaries benchmark observationally equivalent to a pooling equilibrium at d^*. Despite affecting some results - namely welfare and the endogenous primaries equilibria - the no-primaries benchmark is orthogonal to the fundamental insight provided by the model. Moreover, the same no-primaries benchmark is used for example by Snyder and Ting (2011). In Serra (2011), instead, the assumption is that, without primaries, the probability of picking a valent politician is zero. My choice of benchmark reflects my willingness to isolate the effect of the costly signaling and without confounding it with the effect of changing the pool from which politicians are drawn. If I allowed primaries to improve the pool of politicians, the insight of the main theorem would remain unchanged, and there would only be more of an upside for primaries to be welfare improving.[17]

A different no-primaries benchmark would be one where parties exert no control over politicians: for example, one could allow multiple Democratic-leaning candidates to run in the first stage of the general election, followed by a run-off. In such a setting, similar signaling mechanisms can arise, with low valence candidates converging to the expected median voter location, that is $\frac{1}{2}$, and valent candidates choosing more extreme policies. The analysis of such a case, which is reminding of the so called blanket primaries, is left for future work, together with the analysis of other types of primary elections, such as open primaries.

5 Primaries in Both Parties and Endogenous Primaries

In this section I present a generalized version of the model, in which the Republican party is perfectly symmetric to the Democratic party and both parties simultaneously hold a primary election. The consequence of having a double primary is that, on each side, the separating equilibrium is played against an opponent of uncertain location and valence.

The core of the analysis is very similar to the single primary case: a separating equilibrium requires incentive compatibility and that each party voter prefers the valent politician at the more extreme location to the non-valent politician at the more moderate location.

Before stating the result of the double-primary model, it is convenient to define some

[17]In Appendix B I describe a less realistic but theoretically interesting benchmark, in which parties offer a menu of platforms for candidates to choose from. This menu can achieve separation as in the primary election equilibrium, but it distorts the platform of non-valent politicians towards the centre, in order to allow valent candidates to separate with a platform closer to d^*.

new notation: $U_i(d, r, \nu_D, \nu_R)$ is the expected utility for party $i \in \{D, R\}$ from the general election race between a Democratic candidate located at d with valence ν_D and a Republican candidate located at r with valence ν_R. Further denote by $s \equiv \alpha(2-\alpha)$ the probability that each primary election selects a valent candidate and by $\rho \equiv \frac{v}{2\epsilon}$ the double-primary environment analogue of $\tilde{\rho} = \frac{v}{\tilde{r}}$. Thus, the double-primary analogue of Assumption 1 is that $\rho \leq 1$. Finally, the analogue of Condition 1 for the double-primary model is a condition ensuring that:

$$sU_D(d_H^{DP}, r_H^{DP}, v, v) + (1-s)U_D(d_H^{DP}, r^*, v, 0) \geq sU_D(d^*, r_H^{DP}, 0, v) + (1-s)U_D(d^*, r^*, 0, 0)$$

This condition can be rewritten as a lower bound on ρ which depends on α:

Condition 2. $\rho \geq \rho_3(\alpha)$

Derivation. See Appendix. □

The following proposition summarizes the main results of the double-primary model:

Proposition 1. *Assume that $\rho \in [\rho_3(\alpha), 1]$. Then, the double primary game has a unique separating equilibrium satisfying the Intuitive Criterion.*[18] *On the Democratic side, valent politicians separate by proposing platform d_H^{dp} such that:*

$$d_H^{dp} = d^* - \frac{1}{1-\alpha}\left[\frac{2\epsilon - \alpha(2-\alpha)v}{(2-\alpha)}\right], \tag{4}$$

whereas non-valent politicians choose $d_L^{dp} = d^* = \frac{1}{2} - \epsilon$. The locations of the Republican candidates are symmetric to those of the Democratic ones, hence $r_H^{dp} = 1 - d_H^{dp}$ and $r_L^{dp} = 1 - d_L^{dp}$.

Proof. See Appendix. □

As one can see from the expression for d_H^{dp}, there are two opposite effects at work: uncertainty on the opponent's location and uncertainty on the opponent's valence. The former has a multiplier effect on the movement required to separate, which is represented by the $\frac{1}{1-\alpha}$ factor, which was not present in the single primary expression: this strategic complementarity reinforces the polarization effect of primaries. The multiplier effect is increasing in the share of valent politicians. As far as the valence of the opponent is concerned, it has an effect of strategic substitution which decreases the polarizing effect

[18] As I explain in the appendix, the separating equilibrium surviving the Intuitive Criterion is unique, but the Intuitive Criterion does not always kill all pooling equilibria. As a result, there can be pooling equilibria even when Condition 2 is satisfied.

of primaries. It can be noticed that the numerator of the second term of (4) is simply the expected value of the opponent's valence, since $\alpha(2-\alpha) = 1-(1-\alpha)^2$, i.e. the probability that at least one valent politician is drawn by each party. Up to the multiplier effect, therefore, (4) is equivalent to the separation condition in a single primary model with $r = \frac{1}{2} + \epsilon$ and the valence of the incumbent being v with probability α. Notice that compared to the single-primary case where $\tilde{r} = 2\epsilon$, the movement to the left of valent Democratic candidates in the double primary is larger if and only if $\rho \geq \frac{1}{2-\alpha}$.

5.1 Endogenous Primary Choice

The question of whether parties would endogenously choose to hold primaries has so far remained unanswered. In this section I consider what would happen if both parties were to choose, at the beginning of the game, whether to hold a primary or play the no-primary benchmark, which is equivalent to a pooling equilibrium with candidates located at d^*.

The result is that depending on the parameter values, there can be either a no-primary equilibrium (that is an equilibrium in which both parties do not hold primaries), or a double-primary equilibrium (that is one in which both parties endogenously decide to hold a primary). Moreover, it is also possible for an asymmetric equilibrium to arise, in which only one party holds primaries.[19] Whenever asymmetric equilibria exist, there exists also a symmetric equilibrium in mixed strategies, in which each party holds primaries with positive probability.

Intuitively, the no primary equilibrium requires low values of ρ (that is low valence with respect to uncertainty in the location of the median voter), whereas the double-primary equilibrium requires higher values of ρ. The higher α, the higher is the value of ρ necessary to sustain an equilibrium with primaries, since parties particularly benefit from primaries when the pool of potential candidates is bad.

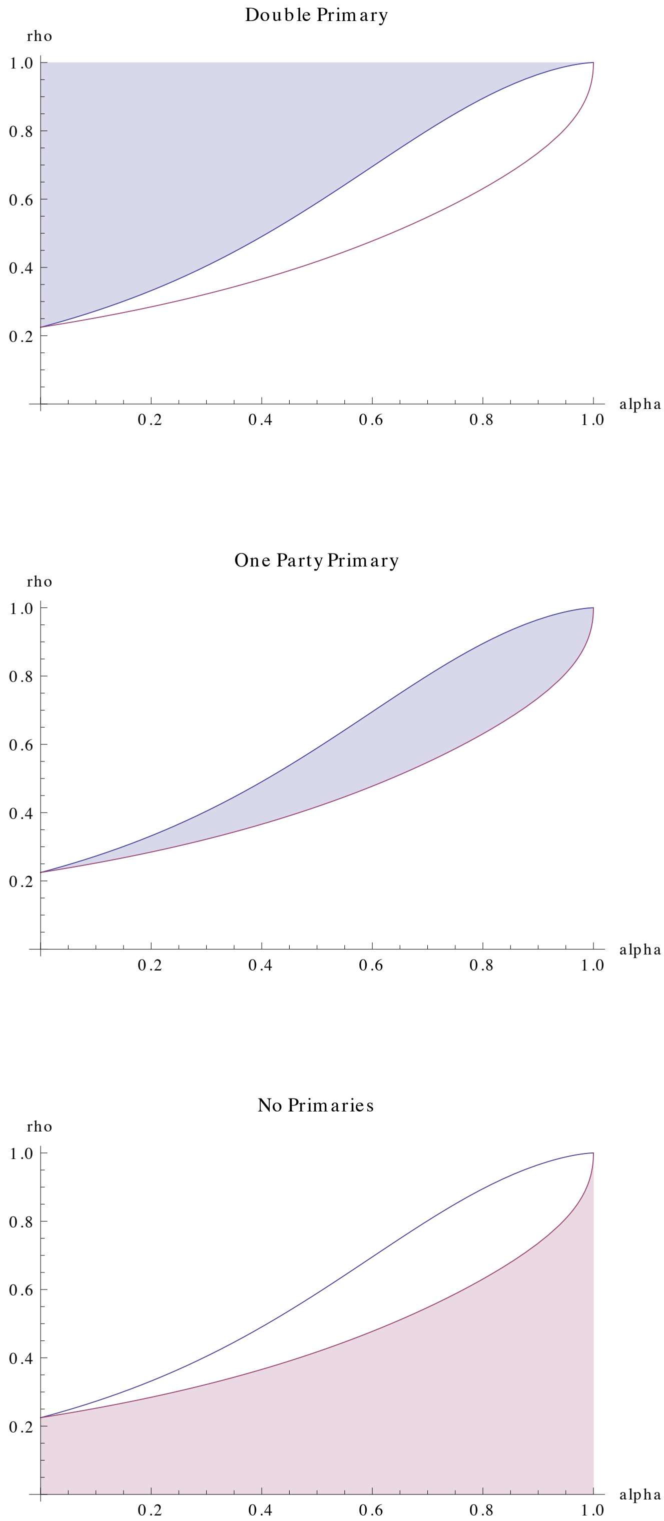
Proposition 2. *The endogenous double-primary game has three potential outcomes: for values of α and ρ such that ρ is low enough (as a function of α), in the unique equilibrium parties choose not to hold primaries. If α is low and ρ is high, on the other hand, the unique equilibrium has parties endogenously choosing to hold primaries. Finally, there is an intermediate region in which either only one party holds primaries, or both hold them with symmetric positive probability.*

Proof. See Appendix. □

[19] It is in fact the case that in some countries, primaries have been held by only one of the major parties, such as the the Partito Democratico in Italy.

Figure 1 summarizes the endogenous primaries model results: as the first plot in 1 shows, in the region north-west of the blue curve, that is when α is low and ρ is high, both parties hold primaries in equilibrium. Intuitively, when α is low and ρ is high, primaries substantially increase the probability of having a valent candidate; moreover, when valence is high relative to the uncertainty in the location of the median voter, having a valent candidate is an important asset for a party. In addition to these two reasons, the policy that a valent politician needs to propose in order to signal his quality is less extreme when α is low. On the other hand, to the south east of the purple curve, in equilibrium both parties choose not to hold primaries. In this region, valence is low with respect to electoral uncertainty and the pool of candidates is good. Therefore, the incremental effect of primaries on the quality of candidates is lower and valence is not high enough to affect the election to a very high extent. On top of that, when α is high the platform needed by valent candidates to signal their quality is more extreme: this makes primaries even less attractive. Interestingly, between the blue and the purple curves in Figure 1, only one party has the incentive to hold primaries in equilibrium. In an asymmetric equilibrium, the party holding primaries is disadvantaged in the general election but selects candidates of higher quality who choose more extreme policies closer to the party bliss point: as a result, the voter of the party holding primaries is better off compared to the voter of the party not holding primaries. Once one party is holding primaries, however, the opposing party prefers to stick to not having primaries in order to enjoy the electoral advantage.

Figure 1: Double Primary Equilibrium Characterization



6 Conclusion

This paper develops a model of signaling in primary elections, in which high quality candidates use more extreme policy platforms to separate from low valence ones. Extremism is thus the result of a signaling effort. Therefore, this paper suggests that policy polarization might happen for reasons independent from the ideology of candidates and party voters.

The result that valent candidates choose more radical policies in primary elections to signal their type is novel and it provides a useful complement to what has been found in existing models of primary elections, in which valence is assumed to be observable when politicians choose their platforms. Depending on the context, for example how well known primary candidates are, or on the type of characteristics captured by valence, one might prefer to think of valence as publicly or privately observable. My model sheds light on the fact that this distinction is very important for the predicted outcome of a primary election.

Moreover, I also show that if parties can choose whether to hold primaries, an equilibrium with primaries is not always sustainable, even if parties are always better off when they both hold primaries than when they both do not hold them. In this context it is possible that only one political party decides to hold primaries in equilibrium.

Future research on this topic could enrich the model of primaries with new factors such as ideological differences within the party or primary elections of varying degrees of openness; moreover, it would be interesting to investigate the implications of the mechanism I describe in my model on the endogenous supply of valent politicians and on the incentives of media to exert effort in revealing the type of candidate before the general election.

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A Proofs

Proof of Lemma 1. Let's consider the decision problem of a Democratic party voter which needs to decide the location of its candidate when facing an opponent located at r and with valence fixed at zero; denote by μ the probability the party assigns to the candidate being valent.

Denoting by $\mathbb{E}U_D(d,\mu)$ the expected utility the party derives from a candidate locating at d and who is believed to be valent with probability μ can be written as:

$$\mathbb{E}U_D(d,\mu) = \mu U_D(d,r,v) + (1-\mu)U_D(d,r,0)$$

Using the fact that $U_D(d,r,\nu) = \pi(d,r,\nu)(\nu - d) + (1-\pi(d,r,\nu))(-r)$, as well as the fact[20] that $\pi(d,r,\nu) = \frac{\tilde{r}-\tilde{d}+v}{4\epsilon}$ we can rewrite the expression as:

$$\mathbb{E}U_D(d,\mu) = -r + \mu\left[\frac{\tilde{r}-\tilde{d}+v}{4\epsilon}(\tilde{r}+\tilde{d}+v)\right] + (1-\mu)\left[\frac{\tilde{r}-\tilde{d}}{4\epsilon}(\tilde{r}+\tilde{d})\right]$$

Differentiating with respect to d yields:

$$\tilde{d}^* = 0 \Leftrightarrow d^* = \frac{1}{2} - \epsilon$$

An analogous derivation can be used to show that d^* is the optimal policy even with uncertainty over the location over the Republican candidate (interpret μ and $1-\mu$ as the probabilities that the Republican candidate is located at some point r' and r respectively) or if there is uncertainty over the valence of the Republican opponent instead of the Democratic candidate. □

Derivation of Condition 1. This condition provides both an existence and a uniqueness result: first, it shows under what conditions a separating equilibrium exists. Moreover, it also finds the conditions under which the Intuitive Criterion removes all equilibria other than the separating one I describe. In this game, as a matter of fact, the Intuitive Criterion alone is not sufficient to remove all pooling equilibria. I start from the uniqueness part by showing how the condition prevents pooling equilibria.

Suppose that candidates are pooling at some point d_p. By the Intuitive Criterion, a valent politician can credibly signal his type by choosing policy d_{pH} which is equilibrium

[20] In order to see this, take $\pi(d,r,v) = \frac{1}{2} + \frac{2\epsilon+r+d+v-1}{4\epsilon}$. This can be rewritten as $\pi(d,r,v) = \frac{r-(1/2-\epsilon)+d-(1/2-\epsilon)+v}{4\epsilon} = \frac{\tilde{r}-\tilde{d}+v}{4\epsilon}$

dominated for the non-valent type:

$$\pi(d_{pH}, r, 0) \leq \frac{1}{2}\pi(d_p, r, 0).$$

Algebrically this delivers:

$$\tilde{d}_{pH} = \tilde{d}_p + \frac{1}{2}(\tilde{r} - \tilde{d}_p).$$

If $U_D(d_{pH}, r, v) \geq U_D(d_p, r, \nu)$, then this is feasible and the pooling equilibrium is destroyed; if $U_D(d_{pH}, r, v) < U_D(d_p, r, \nu)$, this kind of deviation cannot destroy the pooling equilibrium, because even if the signaling is credible, it would not lead to the deviating politician to win the primaries. $U_D(d_{pH}, r, v) < U_D(d_p, r, \nu)$ can be rewritten as

$$[\tilde{d}_p + (\tilde{r} - \tilde{d}_p)\frac{1}{2}]^2 - \tilde{d}_p^2 - (1 - \alpha)(v^2 + 2\tilde{r}v) > 0. \tag{5}$$

I am going to restrict parameters such that this inequality is never satisfied. In order to do that, take (5) and let it hold as equality. This becomes a quadratic equation in $\tilde{d}_p^2$. If this equation has no solution, then (5) is never satisfied and the Intuitive Criterion can be used to deliver a unique equilibrium.

The quadratic equation implied by (5) has no real roots if and only if:

$$\tilde{r}^2 \leq 3(1 - \alpha)(v^2 + 2\tilde{r}v). \tag{6}$$

Dividing through by $\tilde{r}^2$ we get the following quadratic inequality in $\tilde{\rho}$,

$$3(1 - \alpha)\tilde{\rho}^2 + 6(1 - \alpha)\tilde{\rho} - 1 \geq 0$$

which is satisfied for sufficiently large $\tilde{\rho}$, the value of which depends on α. Therefore,

$$\tilde{\rho}_1(\alpha) < \tilde{\rho} < 1.$$

Notice that for some values of α, $\tilde{\rho}_1(\alpha) > 1$, so Condition 1 also implicitly puts an upper bound on α.

The argument for a unique separating equilibrium surviving the Intuitive Criterion is the standard one and it is reported in the proof of Theorem 1.

Finally, Condition 1 also assures that the separating equilibrium with non valent politicians choosing d^* and valent politicians choosing d_H such that $\tilde{d}_H = \frac{\tilde{r}}{(2-\alpha)}$ always exists. In other words, the Democratic party prefers the valent candidate at location d_H

to the non-valent candidate at location d^*. Formally:

$$U_D(d_H, r, v) \geq U_D(d^*, r, 0)$$

Using the fact that $U_D(d_H, r, v) = -r + \pi(d_H, r, v)(d_H - r + v)$ and $U_D(d^*, r, 0) = -r + \pi(d^*, r, v)(d^* - r)$, and using the fact that $\pi(d_H, r, v) = \frac{\tilde{r} - \tilde{d}_H + v}{4\epsilon}$ and [21] $\pi(d^*, r, 0) = \frac{\tilde{r}}{4\epsilon}$, we can rewrite the condition as:

$$\frac{1}{4\epsilon}(\tilde{r} - \tilde{d}_H + v)(\tilde{r} + \tilde{d}_H + v) - r \geq \frac{1}{4\epsilon}\tilde{r}^2$$

which can be rearranged to:

$$d_H \leq \sqrt{(v + \tilde{r})^2 - \tilde{r}^2}.$$

Now, substituting for the separating platform $\tilde{d}_H = \frac{1}{2-\alpha}\tilde{r}$ we obtain

$$\tilde{r}^2 \leq (2 - \alpha)^2(v^2 + 2\tilde{r}v). \tag{7}$$

Since $(2 - \alpha)^2 > 3(1 - \alpha)$, (7) is implied by (6).

□

Proof of Theorem 1. In a separating equilibrium, $w_H = 1 - \frac{\alpha}{2}$ and $w_L = \frac{1-\alpha}{2}$. A separating equilibrium is sustainable if non-valent politicians have no incentive to mimic valent politicians by choosing d_H. Using the fact that:

$$U_c(d_H, v) = w_H \pi(d_H, r, v),$$

incentive compatibility is captured by equation (1) from the main text, which reads:

$$U_c(d_L, 0) \geq U_c(d_H, 0)$$

This can be rewritten as:

$$\frac{1-\alpha}{2}\left[\frac{1}{2} + \frac{1}{4\epsilon}(d_L + r - 1)\right] \geq \left(1 - \frac{\alpha}{2}\right)\left[\frac{1}{2} + \frac{1}{4\epsilon}(d_H + r - 1)\right]$$

yielding

$$d_H \leq \frac{1-\alpha}{2-\alpha}d_L + \frac{1}{2-\alpha}(1 - 2\epsilon - r)$$

In other words, politicians can credibly signal their valence by moving their platform to

[21] Notice that by definition $d_H = d^* - \tilde{d}_H$

the left.

Thanks to the Intuitive Criterion, all platform choices satisfying the above condition will be attributed to the valent type, that is $\nu(p) = v$ for all $p \leq d_H$; since $U_c(d, v)$ is strictly increasing in d, i.e. moving to the left is costly for the general election, valent politicians will choose the most moderate of the separating policies, and therefore we are left with a unique separating equilibrium for each value of d_L, in which the above constraint holds with equality.

In order to pin down the location of d_L, suppose that in a candidate separating equilibrium we had $d_L = d'_L \neq d^* = \frac{1}{2} - \epsilon$. Any deviation to some $\hat{d}_L \in (d'_L, d^*)$ will lead to a certain primary victory whenever the other candidate is a non-valent politician located at d'_L. The reason is that given beliefs on the politician's type, a policy choice closer to d^* gives higher utility to the party, by Lemma 1. In terms of beliefs, the beliefs at d'_L are already the worst possible ones, since they assign probability 0 to being valent. Therefore, no matter what equilibrium beliefs the party assigns to policies in (d'_L, d^*), the party prefers the politician deviating to $\hat{d}_L$ to the politician at d'_L.

When $d_L = d^*$, deviations to $\hat{d}_L \leq d_H$ are prevented by (1), whereas deviations to any $\hat{d}_L \geq d_H$ are prevented by appropriate out-of equilibrium beliefs. One such example is $\nu(p) = 0$ for all $p > d_H$ and $\nu(p) = 1$ for $p \leq d_H$.

In terms of pooling equilibria, Condition 1 ensures that taken any possible pooling equilibrium at d_p, the high type can credibly signal his type by choosing d_H such that $\tilde{d}_H^2 \geq \frac{\bar{r}+\tilde{r}}{2}$. In other words, Condition 1 ensures that for all d_p:

$$U_D(d_H, v, r) \geq U_D(d_p, \nu, r)$$

It follows that under Condition 1, the Intuitive Criterion removes all equilibria other than the separating equilibrium I focus on. □

Proof of Corollaries to Theorem 1. In the following I present the proofs of the corollaries of Theorem 1:

Corollary 1: the result is immediate from inspection of $\tilde{d}_H = \frac{\tilde{r}}{2-\alpha}$, where $\tilde{r} = r - \frac{1}{2} + \epsilon$; notice that an increase of $\tilde{d}_H = \frac{1}{2} - \epsilon - d_H$ means that d_H decreases, i.e. moves to the left. Differentiating yields $\frac{\partial \tilde{d}_H}{\partial \alpha} > 0$, $\frac{\partial \tilde{d}_H}{\partial r} > 0$ and concerning ϵ, we can write $d_H = \left(\frac{1}{2} - \epsilon\right) \frac{3-\alpha}{2-\alpha} - \frac{r}{2-\alpha}$ and see that $\frac{\partial d_H}{\partial \epsilon} < 0$.

Corollary 2: The probability for the Democratic party to win the election under a separating equilibrium is:

$$\Pi_{pr} = [1-(1-\alpha)^2]\pi(d_H, v, r) + (1-\alpha)^2\pi(d^*, 0, r) = \alpha(2-\alpha)\frac{\tilde{r} - \tilde{d}_H + v}{4\epsilon} + (1-\alpha)^2\frac{\tilde{r}}{4\epsilon}$$

Without primaries, the probability that the Democratic party wins is instead:

$$\Pi_{npr} = \alpha\pi(d^*, v, r) + (1-\alpha)\pi(d^*, 0, r) = \alpha\frac{\tilde{r}+v}{4\epsilon} + (1-\alpha)\frac{\tilde{r}}{4\epsilon}$$

Some algebra shows that

$$\Pi_{pr} \leq \Pi_{npr} \iff \tilde{\rho} \leq \frac{1}{1-\alpha}$$

which always holds under Assumption 1.

Corollary 3: the probability of winning the general election conditional on winning the primaries is $\pi(d_H, v, r) = \frac{\tilde{r}-\tilde{d}_H+v}{4\epsilon}$ for valent candidates and $\pi(d^*, 0, r) = \frac{\tilde{r}}{4\epsilon}$ for non-valent ones:

$$\pi(d_H, v, r) \geq \pi(d^*, 0, r) \iff \tilde{d}_H \leq v \iff \tilde{\rho} \geq \frac{1}{2-\alpha}.$$

As far as the unconditional probability is concerned, the comparison is

$$w_H\pi(d_H, v, r) \geq \frac{1}{2}\pi(d^*, 0, r) \iff w_H(\tilde{r} - \tilde{d}_H + v) \geq \frac{1}{2}(\tilde{r}+v) \iff \tilde{\rho} \geq \frac{\alpha}{1-\alpha}.$$

Corollary 4: Comparing the expected utility of the Democratic party when primaries are used as opposed to when no primaries are used yields the following:

$$\alpha(2-\alpha)[(\tilde{r}+v)^2 - \tilde{d}_H^2] + (1-\alpha)^2\tilde{r}^2 \geq \alpha(\tilde{r}+v)^2 + (1-\alpha)\tilde{r}^2$$

This can be rearranged as $(1-\alpha)(2-\alpha)(\tilde{\rho}^2 + 2\tilde{\rho}) - 1 \geq 0$. Denoting by $K \equiv \frac{1}{(1-\alpha)(2-\alpha)}$, the condition is satisfied as long as $\tilde{\rho} \geq \sqrt{1+K} - 1 \equiv \tilde{\rho}_2(\alpha)$. Notice that $\tilde{\rho}_2(\alpha) < 1$ requires α to be sufficiently low. In other words, under the restriction of Assumption 1, primaries can benefit the Democratic party as long as α is sufficiently low.

Corollary 5: there are two cases to consider. When $\tilde{d}_H > v$, all types of median voter srtictly prefer the non-valent candidate at d^* compared to the valent candidate located at d_H: this happens when $\tilde{d}_H > v$. It is therefore trivial that in this case, no median voter type can be made better off by the adoption of a Democratic primary. If instead $v > \tilde{d}_H$, the valent Democratic candidate is preferred to the non-valent one by at least some voters. However, even in this case all median voter types are worse off. In order to see that it is sufficient to check whether $m = d^*$ can be better off, since all voters with $m \in [d^*, d^* + (v - \tilde{d}_H)]$ always vote for d, and therefore their expected utility from the

Democratic candidates is strictly decreasing in m. The expected utility of the voter at d^* when there are no primaries is αv, whereas when the Democratic party adopts primaries the utility becomes $(1-(1-\alpha)^2)(v-\tilde{d}_H)$. However,

$$(1-(1-\alpha)^2)(v-\tilde{d}_H) \geq \alpha v \iff \tilde{\rho} \geq \frac{1}{1-\alpha}$$

which is ruled out by Assumption 1. In other words, even the leftmost median voter type is made worse off by the introduction of the Democratic primary. Notice that the condition $\tilde{\rho} \geq 1/(1-\alpha)$ is the same according to which the Democratic party's probability of winning the election increases when adopting the primary election. For all voters to the right of $d^* + (v - \tilde{d}_H)$, the Democratic candidate in a separating equilibrium is worse than the non valent one. Therefore, they would be at least as well off if the Democratic candidate always nominated a non-valent candidate at d^*, and therefore they would also be at least as well off under the no-primary equilibrium. The preference turns out to be strict for all except that for the median voter types to the right of $d^* + \frac{\tilde{r}+v}{2}$, who always choose the Republican candidate independently of whether the Democratic party holds primaries and are therefore indifferent. □

Derivation of Condition 2. This condition is similar to Condition 1 for the single-primary game, with the only difference that it only concentrates on existence of the (unique) separating equilibrium and does not rule out the possibility that pooling equilibria survive the Intuitive Criterion.

In other words, the aim of this condition is to determine the parameter values such that the party voter prefers the valent candidate at d_H^{DP} (or r_H^{DP}) to the non-valent candidate at d^* (or r^*). I base my calculations on the Democratic side, but analogous calculations hold for the Republican side. Since primaries are simultaneous I analyze an interim condition, meaning that when choosing the candidate, the party voter does not know the valence and location of the opponent yet. The expected utility the Democratic party voter gets from a valent candidate at d_H^{DP}, which will henceforth denoted by d_H for ease of notation, is the following:

$$\mathbb{E}_r U_D(d_H, r, v, \nu_R) = sU_D(d_H, r_H, v, v) + (1-s)U_D(d_H, r_L, v, 0)$$

where $s = \alpha(2-\alpha)$ denotes the probability that the Republican primary selects a valent candidate under a separating equilibrium. This can be written, simplifying the notation of $\mathbb{E}_r U_D(d_H, r, v, \nu_R)$ to $U_D(d_H, v)$, as:

$$U_D(d_H, v) = \alpha(2-\alpha)\left[\frac{1}{2}(2\epsilon + \tilde{d}_H^{DP}) - \frac{1}{2}\tilde{d}_H^{DP} + v\right] + (1-\alpha)^2\left[\left(\frac{1}{2} - \frac{\tilde{d}_H^{DP} - v}{4\epsilon}\right)\left(2\epsilon + \tilde{d}_H^{DP} + v\right)\right] - r^*$$

Following the same logic, the expected utility from choosing a non valent candidate at d^* is:

$$U_D(d^*, 0) = \alpha(2-\alpha)\left[\left(\frac{1}{2} + \frac{\tilde{d}_H^{DP} - v}{4\epsilon}\right)2\epsilon + \left(\frac{1}{2} - \frac{\tilde{d}_H^{DP} - v}{4\epsilon}\right)(v - \tilde{d}_H^{DP})\right] + (1-\alpha)^2\left[\frac{1}{2}2\epsilon\right] - r^*$$

The former can be simplified to (I drop the exponent DP from the notation for the d location):

$$U_D(d_H, v) = -\frac{1}{2} + v - (1-\alpha)^2\frac{\tilde{d}_H^2}{4\epsilon} + (1-\alpha)^2\frac{v^2}{4\epsilon}$$

whereas the utility from the low valence candidate can be written as:

$$U_D(d^*, 0) = -\frac{1}{2} + \alpha(2-\alpha)\frac{\tilde{d}_H^2}{4\epsilon} + \alpha(2-\alpha)\frac{v^2}{4\epsilon} - 2\alpha(2-\alpha)\frac{v\tilde{d}_H}{4\epsilon}$$

In order for separation to be feasible, it has to hold that $U_D(d_H, v) \geq U_D(d^*, 0)$, which requires:

$$v + [2(1-\alpha)^2 - 1]\frac{v^2}{4\epsilon} - \frac{\tilde{d}_H^2}{4\epsilon} + 2\alpha(2-\alpha)\frac{v\tilde{d}_H}{4\epsilon} \geq 0$$

Using the fact that $\tilde{d}_H = \frac{2\epsilon}{(1-\alpha)(2-\alpha)} - \frac{\alpha}{1-\alpha}v$ one obtains the following expression in terms of $\rho \equiv \frac{v}{2\epsilon}$:

$$\underbrace{\left[2(1-\alpha)^2 - 1 - \frac{\alpha^2}{(1-\alpha)^2} - \frac{2\alpha^2(2-\alpha)}{1-\alpha}\right]}_{A(\alpha)}\rho^2 + 2\underbrace{\left[1 + \frac{\alpha}{(2-\alpha)(1-\alpha)^2} + \frac{\alpha}{1-\alpha}\right]}_{B(\alpha)}\rho +$$

$$-\underbrace{\frac{1}{(1-\alpha)^2(2-\alpha)^2}}_{C(\alpha)} \geq 0$$

Solving this quadratic equation we get a threshold $0 < \rho_3(\alpha) < 1$ such that the inequality is satisfied for $\rho \geq \rho_3(\alpha)$. Moreover, it can be checked numerically that $\rho_3(\alpha)$ is increasing in α.

Notice that unlike Condition 1, for ease of exposition given the increased complexity of the double-primary game, Condition 2 does not address the point of uniqueness. In other words, it cannot be ruled out that for some parameter values there might be pooling equilibria surviving the Intuitive Criterion. Additional details on this issue are available upon request. □

Proof of Proposition 1. The proof is in many aspects analogous to the one for the single-primary game. In particular, the argument according to which, in a separating equilibrium, non-valent politicians respectively locate at d^* and $r^* = 1 - d^*$ is exactly the same as that in the single-primary game. [22]

Here I will therefore focus on the incentive compatibility constraint, which determines the separating policy for the valent politician. The constraint for the Democratic side of the game, canceling out $\frac{1}{4\epsilon}$, writes:

$$w_H((1-s)\pi(d_H, r_L, 0, 0) + s\pi(d_H, r_H, 0, v)) \leq w_L((1-s)\pi(d_L, r_L, 0, 0) + s\pi(d_L, r_H, 0, v))$$

where $s = \alpha(2-\alpha)$ is the probability that the (Republican) primary election selects a valent candidate when a separating equilibrium is played. The incentive compatibility constraint can be rewritten in the following way:

$$\left(1 - \frac{\alpha}{2}\right)\left[(1-\alpha)^2(2\epsilon - 1 + d_H + r_L) + \alpha(2-\alpha)(2\epsilon - 1 + d_H + r_H - v)\right] \leq$$
$$\leq \frac{1-\alpha}{2}\left[(1-\alpha)^2(2\epsilon - 1 + d_L + r_L) + \alpha(2-\alpha)(2\epsilon - 1 + d_L + r_H - v)\right].$$

The constraint for the Republican candidate is very similar:

$$\left(1 - \frac{\alpha}{2}\right)\left[(1-\alpha)^2(2\epsilon + 1 - d_L - r_H) + \alpha(2-\alpha)(2\epsilon + 1 - d_H - r_H - v)\right] \leq$$
$$\leq \frac{1-\alpha}{2}\left[(1-\alpha)^2(2\epsilon + 1 - d_L - r_L) + \alpha(2-\alpha)(2\epsilon + 1 - d_H - r_L - v)\right].$$

Letting the constraints hold with equality and substituting the second into the first yields the following expression:

$$d_H = \frac{1+\alpha^3 - 2\alpha^2}{(2-\alpha)(1-\alpha)(1+\alpha)} d_L - \frac{1}{(2-\alpha)(1+\alpha)} r_L + \frac{1}{(2-\alpha)(1+\alpha)} - \frac{2\epsilon}{(2-\alpha)(1-\alpha)} + \frac{\alpha}{1-\alpha} v$$

However, notice that in any separating equilibrium, it has to be that $d_L = \frac{1}{2} - \epsilon$ and

[22] It relies on the fact that Lemma 1 does not change if there is uncertainty in location and valence and on the fact that if a non valent Democratic candidate were to locate at $\hat{d} \neq d^*$ (and analogously for a Republican candidate), then a deviation to any location closer to d^* would be profitable, because beliefs over the candidate's type cannot be worse than $Pr(\nu = v) = 0$ and the party strictly prefers locations closer to d^*.

$r_L = \frac{1}{2} + \epsilon$. Substituting for these values in the expression above yields:

$$\frac{1}{2} - \epsilon - d_H = \frac{2\epsilon}{(2-\alpha)(1-\alpha)} - \frac{\alpha}{1-\alpha}v$$

and since by definition $\frac{1}{2} - \epsilon - d_H^{DP} = \tilde{d}_H^{DP}$, one obtains the separating condition stated in the theorem, that is:

$$d_H^{dp} = d^* - \frac{1}{1-\alpha}\left[\frac{2\epsilon - \alpha(2-\alpha)v}{(2-\alpha)}\right]$$

□

Proof of Proposition 2. Given that the characterization of equilibria involves rather complicated expressions, I will derive the expected utility expressions and then plot them to show the three different regions of equilibria in the (α, ρ) space. I begin by finding the expected utility for each party (since the equilibrium is symmetric, expected utility is the same for both parties) in the no-primary equilibrium and in the double primary equilibrium. $U(P, P)$ denotes the utility from the double-primary equilibrium and $U(NP, NP)$ the utility from the no-primary equilibrium. The utilities are written in terms of the Democratic party, but given symmetry analogous expressions hold for the Republican party. Denote by $r^* \equiv \frac{1}{2} + \epsilon$:

$$\begin{aligned} U(NP, NP) = & - r^* + \alpha^2\left[\frac{1}{2}2\epsilon + v\right] + (1-\alpha)^2\left[\frac{1}{2}2\epsilon\right] + \\ & + \alpha(1-\alpha)\left[\left(\frac{1}{2} + \frac{v}{4\epsilon}\right)(2\epsilon + v)\right] + \alpha(1-\alpha)\left[\left(\frac{1}{2} + \frac{v}{4\epsilon}\right)v + \left(\frac{1}{2} - \frac{v}{4\epsilon}2\epsilon\right)\right] \end{aligned}$$

Factoring out ϵ and noting that $-r^* + \epsilon = \frac{1}{2}$ this can simplified to yield: $-\frac{1}{2} + [\alpha^2 + 2\alpha(1-\alpha)(\frac{1}{2} + \frac{v}{4\epsilon})]v$ which can be further simplified to yield:

$$U(NP, NP) = -\frac{1}{2} + \alpha v + 2\alpha(1-\alpha)\frac{v^2}{4\epsilon}$$

Notice that this can also be rearranged as the following:

$$U(NP, NP) = -\frac{1}{2} + [1 - (1-\alpha)^2]v - 2\alpha(1-\alpha)\left(\frac{1}{2} - \frac{v}{4\epsilon}\right)v.$$

It can be shown that any symmetric equilibrium provides an expected utility of this form.

Next I derive the expression for the expected utility in a double-primary equilibrium:

$$\begin{aligned}U(P,P) =& - r^* + \alpha^2(2-\alpha)^2\left[\frac{1}{2}(-\tilde{d}_H^{DP}) + \frac{1}{2}(2\epsilon + \tilde{d}_H^{DP}) + v\right] + (1-\alpha)^4\left[\frac{1}{2}2\epsilon\right] + \\ &+ \alpha(2-\alpha)(1-\alpha)^2\left[\left(\frac{1}{2} + \frac{\tilde{d}_H^{DP} - v}{4\epsilon}\right)2\epsilon + \left(\frac{1}{2} - \frac{\tilde{d}_H^{DP} - v}{4\epsilon}\right)(-\tilde{d}_H^{DP} + v)\right] + \\ &+ (1-\alpha)^2\alpha(2-\alpha)\left[\left(\frac{1}{2} - \frac{\tilde{d}_H^{DP} - v}{4\epsilon}\right)(2\epsilon + \tilde{d}_H^{DP} + v)\right]\end{aligned}$$

This can be simplified to the following expression:

$$U(P,P) = -\frac{1}{2} + [1-(1-\alpha)^4]v - 2\alpha(2-\alpha)(1-\alpha)^2\left(\frac{1}{2} + \frac{\tilde{d}_H^{DP} - v}{4\epsilon}\right)v$$

and further to:

$$U(P,P) = -\frac{1}{2} + [1-(1-\alpha)^4 - \alpha(1-\alpha)((1-\alpha)(2-\alpha)+1)]v + 2[\alpha(2-\alpha)(1-\alpha)]\frac{v^2}{4\epsilon}$$

where I used the fact that $\tilde{d}_H^{DP} = \frac{2\epsilon}{(1-\alpha)(2-\alpha)} - \frac{\alpha}{1-\alpha}v$. I now calculate the payoff from a deviation out of the no-primary and double-primary equilibrium. The first reads:

$$\begin{aligned}U(P,NP) =& - r^* + (1-\alpha)^3\left(\frac{1}{2}2\epsilon\right) + \alpha^2(2-\alpha)\left[v + \left(\frac{1}{2} - \frac{\tilde{d}_H}{4\epsilon}\right)(2\epsilon + \tilde{d}_H)\right] + \\ &+ \alpha(1-\alpha)^2\left[\left(\frac{1}{2} + \frac{v}{4\epsilon}\right)v + \left(\frac{1}{2} - \frac{v}{4\epsilon}\right)2\epsilon\right] + \\ &+ \alpha(2-\alpha)(1-\alpha)\left[\left(\frac{1}{2} - \frac{\tilde{d}_H - v}{4\epsilon}\right)(2\epsilon + \tilde{d}_H + v)\right]\end{aligned}$$

which can be simplified to:

$$U(P,NP) = -\frac{1}{2} + \alpha(1-\alpha)^2\frac{v^2}{4\epsilon} + \alpha^2(2-\alpha)\left[v - \frac{\tilde{d}_H^2}{4\epsilon}\right] + \alpha(2-\alpha)(1-\alpha)\left[v + \frac{v^2}{4\epsilon} - \frac{\tilde{d}_H^2}{4\epsilon}\right]$$

and finally to:

$$U(P,NP) = -\frac{1}{2} + \alpha\left[(2-\alpha) + \frac{\alpha}{2-\alpha}\right]v + \alpha\left[(1-\alpha)(3-2\alpha) - \frac{\alpha^2}{2-\alpha}\right]\frac{v^2}{4\epsilon} - \frac{\alpha}{2-\alpha}\epsilon,$$

using the fact that when deviating out of a no-primary equilibrium, the separating policy is such that $\tilde{d}_H = \frac{2\epsilon - \alpha v}{2-\alpha}$. Finally, the utility from a deviation out of the primary equilibrium can be written as:

$$U(NP,P) = -r^* + (1-\alpha)^3\left[\frac{1}{2}2\epsilon\right] + \alpha^2(2-\alpha)\left[\left(\frac{1}{2}-\frac{\tilde{d}_H^{DP}}{4\epsilon}\right)(-\tilde{d}_H) + \left(\frac{1}{2}+\frac{\tilde{d}_H^{DP}}{4\epsilon}\right)2\epsilon + v\right] +$$
$$+\alpha(1-\alpha)^2\left[\left(\frac{1}{2}+\frac{v}{4\epsilon}\right)(2\epsilon+v)\right]$$
$$+\alpha(1-\alpha)(2-\alpha)\left[\left(\frac{1}{2}-\frac{\tilde{d}_H^{DP}-v}{4\epsilon}\right)(-\tilde{d}_H^{DP}+v) + \left(\frac{1}{2}+\frac{\tilde{d}_H^{DP}-v}{4\epsilon}\right)2\epsilon\right]$$

This can be simplified to get:

$$-\frac{1}{2}+\alpha^2(2-\alpha)\left[\frac{(\tilde{d}^{DP})^2_H}{4\epsilon}+v\right]+\alpha(1-\alpha)^2\left[\frac{v}{2}+\frac{v}{2}+\frac{v^2}{4\epsilon}\right]+\alpha(1-\alpha)(2-\alpha)\left[\frac{(\tilde{d}_H^{DP})^2}{4\epsilon}-\frac{v\tilde{d}_H^{DP}}{4\epsilon}-\frac{v\tilde{d}_H^{DP}}{4\epsilon}+\frac{v^2}{4\epsilon}\right]$$

which can further simplified, using $\tilde{d}_H^{DP} = \frac{2\epsilon}{(1-\alpha)(2-\alpha)} - \frac{\alpha}{1-\alpha}v$ to yield:

$$-\frac{1}{2}+\left[\frac{\alpha(1-\alpha)(3-2\alpha)(1-\alpha)^2+2\alpha^2(2-\alpha)(1-\alpha)^2+\alpha^3(2-\alpha)}{(1-\alpha)^2}\right]\frac{v^2}{4\epsilon}-\frac{\alpha^2}{(1-\alpha)^2}v+\frac{\alpha}{(1-\alpha)^2(2-\alpha)}\epsilon$$

The next step involves checking for the conditions under which a no-primary and a primary equilibrium exist: a no-primary equilibrium exists if and only if $U(NP,NP) \geq U(P,NP)$; dividing both sides by 2ϵ and $\rho \equiv \frac{v}{2\epsilon}$ one obtains:

$$\alpha\rho + \alpha(1-\alpha)\rho^2 \geq \alpha\left[(2-\alpha)+\frac{\alpha}{2-\alpha}\right]\rho + \frac{\alpha}{2}\left[(1-\alpha)(3-2\alpha)-\frac{\alpha^2}{2-\alpha}\right]\rho^2 - \frac{\alpha}{2(2-\alpha)}$$

which can be simplified to:

$$\rho^2[(1-\alpha)(2-\alpha)(2\alpha-1)+\alpha^2] + 2\rho[\alpha(2-\alpha)-2] + 1 \geq 0$$

From the solution of this quadratic equation it can be seen that a no-primary equilibrium exists if and only if ρ is sufficiently small, with the threshold depending on α. In particular, for α sufficiently large a no-primary equilibrium always exists. I will now move on to the double-primary equilibrium. Dividing the expressions for $U(P,P)$ and $U(NP,P)$ by 2ϵ, the condition $U(P,P) - U(NP,P) \geq 0$ can be rewritten as:

$$\left[\alpha(2-\alpha)(1-\alpha) - \frac{\alpha(1-\alpha)^3(3-2\alpha)+2\alpha^2(1-\alpha)^2(2-\alpha)+\alpha^3(2-\alpha)}{2(1-\alpha)^2}\right]\rho^2 +$$
$$+\left[1-(1-\alpha)^4-\alpha(1-\alpha)[(1-\alpha)(2-\alpha)+1]+\frac{\alpha^2}{(1-\alpha)^2}\right]\rho - \frac{\alpha}{2(2-\alpha)(1-\alpha)^2} \geq 0$$

Solving this quadratic equation yields another threshold on ρ, depending on α, above which a double-primary equilibrium is sustainable. A numerical exploration shows that this threshold is, for any $\alpha \in [0,1]$, higher than the one below which a no-primary equilibrium is sustainable. This means that there exist values of ρ for which a symmetric pure strategy equilibrium does not exist. In this interval, two types of equilibria are possible: a symmetric equilibrium in mixed strategies (which I will not characterize) or asymmetric equilibria in which one party holds the primary and the other party doesn't.

□

B Optimal Candidate Screening

In this section I describe the selection of politicians taking place if parties could screen candidates by offering a menu of policy platforms. The intuition is the same as that applying to the signaling model described in the paragraphs above, but with one important difference. Since parties can choose the screening policies before drawing the two potential candidates, they can choose the utility maximizing menu of policies, which can be either a pooling or a separating one. Moreover, the pair of separating policies are not the same as those described in the signaling model. The reason is the following: in the signaling model of primaries, candidates compete, and party voters are not able to commit not to vote for a candidate proposing a platform that gives them a higher utility. Therefore, in a separating equilibrium non-valent candidates can never propose platforms other than d^*. In the screening model, on the other hand, the party is able to distort the policy offered to low valence politicians, making it more moderate in order to make the separating policy offered to high valence politicians also more moderate. This can be summarized in the following proposition:

Proposition 3. *The candidate screening game has a unique equilibrium:*
if $\tilde{\rho} < \tilde{\rho}_{scr}(\alpha)$*, then parties offer a pooling menu with* $d = d^*$*; if* $\tilde{\rho} \geq \tilde{\rho}_{scr}(\alpha)$*, then parties offer a separating menu of policies* (d_L^{scr}, d_H^{scr})*, with* $d_L^{scr} \geq d^*$ *and* d_H^{scr} *satisfying the same incentive compatibility constraint as in the signaling game.*

Proof. Assume that the screening party wants to implement a separating equilibrium. Then it has to maximize the following expression:

$$(1-\alpha)^2 \left[\frac{1}{4\epsilon}(2\epsilon - 1 + d_L + r)(r - d_L) - r \right] + [1-(1-\alpha)^2] \left\{ \frac{1}{4\epsilon}(2\epsilon - 1 + d_H + r + v)(r - d_H + v) \right\}$$

subject to

$$d_H = \frac{1-\alpha}{2-\alpha} d_L + \frac{1}{2-\alpha}(1 - r - 2\epsilon)$$

which becomes

$$(1-\alpha)^2\left[\frac{1}{4\epsilon}(2\epsilon-1+d_L+r)(r-d_L)-r\right]+[1-(1-\alpha)^2]\times$$
$$\times\left\{\frac{1}{4\epsilon}(2\epsilon-1+\frac{1-\alpha}{2-\alpha}d_L+\frac{1}{2-\alpha}(1-r-2\epsilon)+r+v)(r-\frac{1-\alpha}{2-\alpha}d_L-\frac{1}{2-\alpha}(1-r-2\epsilon)+v)\right\}.$$

Differentiating with respect to d_L and multiplying by the constant 4ϵ we obtain the following:

$$(1-\alpha)^2[r-d_L-2\epsilon+1-r-d_L]+[1-(1-\alpha)^2]\frac{1-\alpha}{2-\alpha}\times$$
$$\times\left[r+v-\frac{1-\alpha}{2-\alpha}d_L-\frac{1}{2-\alpha}(1-r-2\epsilon)-\left(2\epsilon-1+\frac{1-\alpha}{2-\alpha}d_L+\frac{1}{2-\alpha}(1-r-2\epsilon)+r+v\right)\right]$$

In order to find the maximum (notice that the second order condition is negative), equate the derivative to zero and rearrange to obtain:

$$d_L^{scr}=\frac{1}{2}-\epsilon+\frac{\alpha}{2(1-\alpha)}\left(r-\left(\frac{1}{2}-\epsilon\right)\right)>d^*$$

As one can see, the optimal screening policy for the low type is more moderate than that resulting from the signaling model, which is $d^*=\frac{1}{2}-\epsilon$. Moreover, in order to conclude whether it is optimal to separate types or not, notice that the utility the party derives from playing a separating equilibrium is:

$$-r+\frac{1}{4\epsilon}\left[(2\epsilon-1+r+d_L)(r-d_L)\right](1-\alpha)^2+\frac{1}{4\epsilon}\alpha(2-\alpha)\left[(2\epsilon-1+r+d_H+v)(r-d_H+v)\right]$$

whereas the utility arising from a pooling equilibrium is

$$-r+\alpha\left[\frac{1}{4\epsilon}(r-\frac{1}{2}+\epsilon+v)(r-\frac{1}{2}+\epsilon+v)\right]+\frac{1-\alpha}{4\epsilon}\left[(r-\frac{1}{2}+\epsilon)(r-\frac{1}{2}+\epsilon)\right]$$

This reduces to the inequality:

$$\alpha(2-\alpha)v^2+2\alpha(2-\alpha)\tilde{r}v-\frac{\alpha(4-\alpha)}{4}\tilde{r}^2\geq\alpha v^2+2\alpha v\tilde{r}$$

which finally leads to the condition

$$\frac{v}{\tilde{r}}\equiv\tilde{\rho}\geq\sqrt{1+\frac{4-\alpha}{4(4-\alpha)}}-1\equiv\tilde{\rho}_{scr}(\alpha)$$

under which screening is preferred to pooling. □

C Observability of Valence Off-Equilibrium

The model I presented in the paper assumes that valence is always revealed before the general election. Whereas this is not relevant on the model equilibrium path, it is important as an off-path threat to prevent deviations from non valent politicians. Therefore, when the assumption of valence observability after the primary is relaxed, signaling becomes more costly for valent politicians. However, as long as valence can be exogenously revealed with positive probability, the main insight of the model goes through. In particular, the incentive compatibility constraint becomes the following:

$$\left(\frac{2-\alpha}{2}\right)\left\{z\left[\frac{1}{2}+\frac{1}{4\epsilon}(d_H+r-1)\right]+(1-z)\left[\frac{1}{2}+\frac{1}{4\epsilon}\left(d_H+r+v-1\right)\right]\right\}\leq\frac{1-\alpha}{2}\left\{\frac{1}{2}+\frac{1}{4\epsilon}\left(d_L+r-1\right)\right\}$$

This can be rewritten as:

$$d_H\leq\frac{1}{2-\alpha}(1-2\epsilon-r)+\frac{1-\alpha}{2-\alpha}d_L-(1-z)v$$

As a result, separation requires valent politicians to move to the left by at least:

$$\tilde{d}_H^z=\frac{1}{2-\alpha}\tilde{r}+(1-z)v$$

which is larger than the amount required in the observable valence case and is increasing in v: this makes primaries less attractive compared to the $z=1$ benchmark for both party members and valent politicians. However, the intuition of the model remains unchanged.

D Relaxing the Commitment Assumption

As mentioned in the discussion section earlier, the assumption of commitment to policy can be relaxed. In order to do that, I assume that after the primaries, politicians only have to stick to their policy platform with some probability $1-k$, instead of with probability 1 as in the baseline model. For simplicity, I assume that the Republican candidate does not relocate and is fixed at $r=\frac{1}{2}+\epsilon$ and that when relocation is possible, it only brings the politician back to d^*. In the model modified in this way, the valent politician can separate by choosing policy d_H^c such that $\tilde{d}_H^c$ satisfies:

$$\tilde{d}_H^c\geq\frac{1}{1-k}\frac{1}{2-\alpha}2\epsilon$$

Notice that if on one hand separation now requires a more extreme policy compared to the baseline model, the cost of the more extreme policy required to separate is counter-balanced by the possibility for the valent politician to relocate after the primary election.