Signaling Valence in Primary Elections

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October 2020

Abstract

This paper presents a model of two-stage (primary and general) elections in which primary election candidates differ in terms of a privately observed quality dimension (valence). I show that primary election candidates have the incentive to signal their valence by means of their policy platform choice. There can be two types of separating equilibria in primary elections, with opposite implications concerning the relationship between valence and policy extremism. In an *extremist* equilibrium valent candidates choose more extreme policies than non-valent ones, whereas in a *centrist* equilibrium valent candidates move close to the incumbent from the opposing party. As a result, primary elections can foster the adoption of extremist policies, but they can also have the opposite effect. This result allows the model to also shed light on the circumstances in which party voters are likely to benefit from the introduction of primary elections.

Keywords: Primary Elections; Valence; Policy Extremism; Polarization; Signaling.
JEL Codes: D72; D82.

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1 Introduction

In many democracies, primary elections play an important role in the selection of candidates and policy platforms. In recent years, however, this institution has come under increased scrutiny: at times in which extremism and polarization are seen as some of the biggest threats to the functioning of democracy, many observers have pointed to primary elections as one of the drivers of these phenomenona[1]. The United States, where the increase in polarization has been particularly strong over the last few decades and where primaries are a fundamental feature of the electoral system, have been at the center stage of this debate. The empirical evidence regarding the effect of primary elections on political polarization, however, is very mixed and suggests that primaries might have either a polarizing or a moderating effect on policy platforms, depending on the circumstances[2].

In order to reconcile the mixed empirical evidence on the effect of primary elections on policy extremism, substantial variation in terms of policy outcomes needs to be possible within the same system of primary elections. To address this issue, it is necessary to take a step back and consider the key aspects of voters choices in primary elections. Along with the ideological positioning of a candidate's platform, the other dimension on which many spatial models of elections focus is valence. Following the seminal contribution by Stokes (1963), this concept is used to represent a range of qualities (honesty, integrity and charisma among others) that are independent of the ideological positioning of a candidate and are valued by all voters.

A key issue with valence, given the type of qualities it represents, is its observability. It is often difficult for voters to judge whether a candidate will turn out to be valent. This is especially relevant in primary elections, where the available information on candidates is usually more limited compared to general elections. Building a reputation for valence is in fact a fundamental component of many primary election campaigns.

[1] See for example Fiorina et al. (2006) and Fiorina and Levendusky (2006).

[2] Most studies have focused on the case of the United States, to a good extent due to a matter of data availability: given that primary elections are mandated by law in the US, the only possibility for an empirical investigation is to exploit the staggered introduction of primaries. Using this exogenous variation in the introduction of primaries, Hirano et al. (2010) find little evidence of a positive effect of primary elections on polarization; Cintolesi (2020) instead finds a negative effect. Other studies have instead exploited, rather than the variation between primaries and no primaries, the variation within the system of primary elections, i.e. the so called openness of primaries: McGhee et al. (2014) find that the openness of primary elections has small effects on polarization, whereas Bullock and Clinton (2011) and Gerber and Morton (1998) find significant moderating effects of open primaries. Other studies look at how candidates with different voting records (or policy platforms) fare in primary elections: Brady et al. (2007) find that moderate primary candidates are more likely to lose primary elections, and similarly Burden (2001) and Burden (2004) show that primary election competitiveness leads candidates to diverge further from the center.

What I argue in this paper is that one of the determinants of policy extremism in a system with primary elections is the observability of valence and, relatedly, the different ways in which candidates can use their policy platform to signal their valence to voters. My theory of signaling in primary elections does not only reconcile the mixed empirical evidence on primary elections and policy extremism, but it also sheds light on the relationship between candidate valence and platform choice. This remains to this date an open question in the study of elections, with some studies finding evidence of a negative correlation between valence and extremism, but others pointing in the opposite direction[3].

Hence, the contribution of this paper lies in addressing the question of how candidate valence and policy extremism are related and how this relationship is affected by the presence of primary elections. In order to do so, I build a model in which I analyze the platform choices of candidates with different levels of valence, in an environment where taking office requires winning both the primary and the general election.

As mentioned above, a key ingredient of the model is a friction on the observability of valence. Only candidates know their own valence before the primary election; however, before the general election, valence becomes public information. This captures the increasing availability of information on candidates as the campaign progresses, due to for example media scrutiny[4]. Since voters cannot observe valence prior to the primary election, primary elections involve signaling: another key contribution of my model is thus to explore the issue of signaling in primary elections and investigate its effects on the policy platforms chosen by candidates.

In order to credibly signal their valence in a separating equilibrium, candidates need to deviate from the baseline policy platform of the party. This deviation, though, can be in either direction: candidates can credibly signal valence by being more extreme than the party baseline or by moving towards (or even past) the center of the policy spectrum, closer to the positions of voters from the opposing party. The logic behind

[3]Measuring the valence of political candidates is a difficult task. An often used proxy is political experience: Burden (2004) finds that favored candidates are further away from the median voter. He argues that "in many districts the winning candidate is actually further from the center than the loser, but manages victory on the basis of non-ideological criteria that overwhelm the modest effects of ideological proximity". Ansolabehere et al. (2001) find instead that valent candidates tend to be more moderate. Stone and Simas (2010) find significant divergence between candidates: high quality candidates locate closer to their district's preferences, but their main result is that challengers who choose policy platforms further away from their district receive more votes, suggesting that they possess some quality to make up for their extremism. Relatedly, Fiorina (1974) brings attention the puzzle of what he calls a *flip-flopping representation*, in which *extremes are replaced by extremes*, especially in competitive districts where the Downsian forces leading to convergence should be strongest. Evidence in favor of divergence is also provided by Erikson and Wright (1980) and Erikson and Wright (1997).

[4]My results are robust to considering more general patterns of valence revelation.

these two opposing strategies, however, is the same: a valent primary candidate can credibly signal valence by choosing platforms that non-valent candidates cannot afford to run with. The intuition is simple: choosing an extreme policy platform decreases the probability of winning the general election against the incumbent for any candidate, but it especially hurts non-valent candidates, who cannot use their valence to make up for a more extreme policy platform. Therefore, choosing an extreme enough policy platform is a credible signal of valence. Similarly, being too close to the incumbent condemns a non-valent candidate, but not a valent one, to a certain electoral defeat. Thus, also this *centrist* position can send a credible signal of valence.

These results capture a range of dynamics that are often seen in political competition. The first concerns candidates refusing to adopt compromising platforms. The second and opposing force is that of candidates competing on issues traditionally associated with the opposing party. According to my model, the unifying trait behind both these strategies is the aim of displaying strength. In the former case, the correlation between valence and ideological extremism is positive, in the latter it is negative.

Given this range of possible outcomes, a key question is what are the factors that determine which one will be realized. First of all, a separating equilibrium in which valent candidates signal their quality requires high valence (that is, an electoral environment that is sufficiently candidate-centered), a small prior probability that primary-candidates are valent (which can also be interpreted as a low ability of parties to screen valent candidates), and an incumbent whose policy platform is not too extreme (or similarly, a sufficiently competitive district). Concerning the two possible separating strategies, the ideology of the primary election median voter is an important driver of the outcome: the more extreme the primary election median voter, the more likely candidates are to signal valence by choosing extreme platforms. This feature of the model supports the view that, other things equal, closed primary elections[5] are more likely to select extreme candidates[6]. Furthermore, my model can also be used to understand under which circumstances party voters are most likely to benefit from primaries. This is an important question for several reasons: first, in many democracies, primaries are not mandated by law as in the United States, but parties can choose whether or not to hold them. Second, even in systems with mandated primaries such as the United States, elements of institutional flexibility

[5]Closed primaries are primary elections in which only voters affiliated to that party prior to the election day can vote.

[6]There is empirical evidence of such an effect, see for example Gerber and Morton (1998) and Bullock and Clinton (2011), but, as for example McGhee et al. (2014) and Casas (2019) argue, there are also elements, empirical and theoretical, pointing in the opposite direction. Refer to Footnote 2 for a further discussion.

do persist, allowing for changes in the degree of primary openness and in the influence of party elites over the primary election outcome.

Intuitively, I find that party voters are likely to benefit from primaries when valence is high (or equivalently when valence has a large weight in voters' utility function compared to ideology) and when the fraction of valent politicians is low, such that primaries generate a bigger improvement in the selection of politicians. This last factor could depend on party structure, district characteristics, or on the ability of parties to screen potential candidates. Furthermore, primary voters are more likely to benefit from primaries when the opposing party incumbent is not too extreme: this can be interpreted as primaries being more beneficial in competitive districts rather than in districts in which a party enjoys a strong ideological advantage. Finally, another interesting implication of my model is that primaries can benefit voters independently of their ideology, which goes against the prior that primaries disproportionately empower the more extreme fringes of the electorate: if anything, as I discuss in Section 5, there are elements suggesting that moderate electorates might be more likely to benefit from primaries, given their greater alignment with the interests of office motivated candidates. The paper is structured as follows: Section 2 reviews the related literature; Section 3 and Section 4 present the model and the results respectively; Section 5 provides a welfare analysis and a discussion of equilibrium selection; Section 6 contains a discussion of the institutional implications of the model; finally, Section 7 concludes. In the Appendix, as well as all the proofs, I present the extension of the model allowing both parties to hold primaries. Furthermore, I also relax some important assumptions concerning the distribution of the median voter and the functional form of voters' preferences.

2 Related Literature

My paper is mostly related to two streams of theoretical literature: that on electoral competition with valence and that on primary elections. There are several theoretical papers on electoral competition with valence. Groseclose (2001) and Aragones and Palfrey (2002) mainly deal with the question of equilibrium existence: in both of these models, valent candidates choose more moderate policies than non-valent ones.

Adams and Merrill III (2008) constitutes one of the seminal papers in the literature on spatial elections with primaries. In their model, valence has two components: one is common knowledge and party-specific, the other is unobservable even for candidates at the moment of choosing policy-platforms. The possibility of signaling is thus shut down, unlike in my work. In their model, candidates of parties with a valence advantage always choose more extreme platforms[7], whereas my model allows for both positive and negative correlation between valence and extremism.

Another closely related paper is Hummel (2013). In his model, valence is common knowledge. Moreover, out of the two primary candidates, one is always valent and the other one always non-valent: therefore, valent primary candidates have all the bargaining power and choose more moderate policies in primary elections, in order to be more electable in the general election. In this respect, my model shows that introducing competition among valent primary candidates and making valence unobservable to primary voters can reverse his result.

Concerning the observability of valence, there are several papers that move away from the assumption of full observability: in Snyder and Ting (2011), valence can be revealed by either the primary or the general election campaign, but policy platforms are fixed, unlike in my model. Also in Kartik and McAfee (2007), valence/character is unobservable, creating a positive correlation between valence and extremism, but only non-valent candidates choose policy platforms strategically, unlike in my model. Bernhardt et al. (2011) develop a dynamic model of elections in which, just like in my model, the valence of incumbents is observable whereas the valence of challengers is not. In their model, the correlation between valence and extremism is negative for first-term office holders, but it becomes positive as tenure increases, due to selection through rounds of elections. Casas (2019) assumes that policy preferences are not observable, whereas valence is. In his model, costly endogenous party affiliation leads to a positive correlation between valence and distance from the party mainstream. Finally, Boleslavsky and Cotton (2015) do not study primaries, but in their model the extremeness of the policies chosen by parties increases with the informativeness of the general election campaign.

Concerning primaries as an institution, Serra (2011) considers the incentives for party elites to hold primaries: the key trade-off is between the valence benefits from an expanded pool of nominees and the costs due to ideological differences with party voters[8]. The choice of elites on whether to hold primaries is also present in Slough et al. (2020), who build a repeated model of elections: in their model, primaries are held when polarization is high and they are preferred by disadvantaged parties.

A positive correlation between valence and extremism also appears in several models

[7]In their model, primary election voters vote naively, ignoring electability.

[8]In his model, valence is observable before primary elections and there is no uncertainty over the location of the general election median voter. Serra (2013) considers a noisy selection of politicians through primaries.

with endogenous valence (i.e. valence resulting from players' effort or choices). In Eguia and Giovannoni (2019), a disadvantaged party can tactically choose an extreme platform in order to occupy an ideological space for future elections. In Carrillo and Castanheira (2008), choosing an extreme policy platform acts as a commitment device to invest in valence; similarly, in Serra (2010), candidates can invest in valence to make up for their extreme policy platforms[9]; finally, in Crutzen et al. (2009), primaries provide an incentive to candidates to exert effort, with a signaling component common to my model [10].

3 The Model

Two political parties, L and R (standing for left and right respectively), compete in an election. Party R enters the election with a pre-established candidate, for example an incumbent; the L party instead selects a candidate through a primary election. Two pre-candidates (I use this term to distinguish primary election candidates from general election candidates) take part in the L party primary election[11].

A policy platform is a point on the real line, $[-\infty, +\infty]$. The R party candidate is located at $r \geq 0$ and may be thought of as the incumbent candidate. Voters have single-peaked policy preferences with bliss-points distributed on the real line. The bliss-point of the general election median voter, denoted by μ, is uniformly distributed on the $[-b, b]$ interval[12]. Some voters are party voters, that is they also vote in the primary election of the L party, the challenging party. The set of party voters is exogenously given and I assume that L party voters have bliss-points weakly to the left of $-b$. In particular, the median L party voter's bliss point is $m \leq -b$. In other words, the bliss-point of the general election median voter is always to the right of the bliss-points of L party voters. These assumptions simplify the analysis of voting in the primary election, but they are

[9] A positive correlation between charisma and extremism can also be found in Serra (2018): candidates are policy motivated, and therefore charismatic candidates can afford to choose more extreme policy platforms.

[10] There are a number of other models of primary elections that for brevity I did not cover in the main text: Owen and Grofman (2006) study a two-stage spatial model of elections; Grofman et al. (2019) consider both closed and open primaries with candidates of potentially different observable valence and show that primaries benefit the party closer to the general population median; Takayama (2014) presents a model with observable valence in which primaries lead to policy moderation; Serra (2015) shows that policies can converge to the median despite the presence of primaries; Meirowitz (2005) presents a model in which primaries allow candidates to learn the preferences of voters. Finally, Hummel (2010) and Agranov (2016) focus on the flip-flopping of candidates between the primary and the general election, which is not part of my model, since politicians commit to their platform.

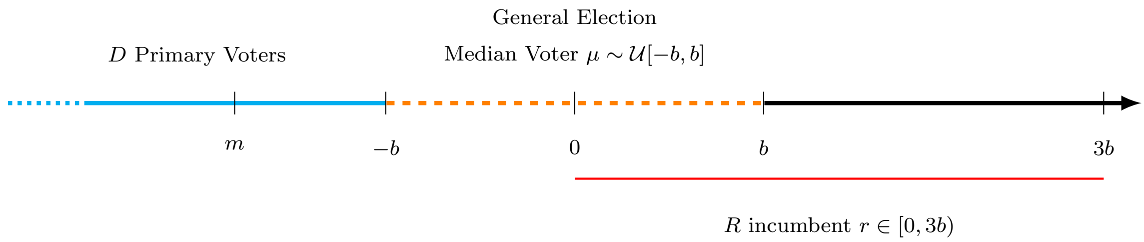
[11] In Appendix B I also consider the case of both parties selecting a candidate through primary elections.

[12] This assumption allows me to work with closed forms, but it does not qualitatively drive the results of the model. In Appendix D I consider a more general distribution function for the general election median voter and show that the main results go through.

not necessary for the model results to hold[13].

Assumption 1. *All primary election voters have bliss-points contained in* $(-\infty, -b]$.

Figure 1: Players Bliss-Points and Platform of R Incumbent



The utility a voter with bliss point x derives from the implementation of a given policy-platform l takes the following additively separable linear form[14]:

$$u_x(l, \theta) = -|l - x| + v(\theta) \tag{1}$$

where $v(\theta)$ indicates the valence of the candidate proposing policy l, which depends on her type $\theta \in \{A, D\}$. In particular:

$$v(\theta) = \begin{cases} v \text{ if } \theta = A \\ 0 \text{ if } \theta = D \end{cases} \tag{2}$$

A fraction α of candidates is of type A, standing for advantaged, and the remaining $1-\alpha$ is of type D, i.e. disadvantaged. For simplicity, I fix the value of party R candidate's valence to $v_r = v$; this captures the incumbency advantage provided by the name recognition, the status of insider and the campaign funding opportunities that play in the incumbent's favor. In addition to that, non-valent incumbents are sometimes removed by a primary challenge[15]. I assume that pre-candidates know their valence level before the primary election, whereas voters can only observe it before the general election or, as we will see, infer it through candidates signaling in the primary election. The key insights of the model would also emerge under a more general setup in which valence is revealed, with

[13] In particular, I could allow for an overlap between the set of primary voters and the support of the general election median voter.

[14] The choice of a linear policy loss function is due to tractability reasons and is common to most models of elections with candidates differing in quality, as see for example Aragones and Palfrey (2002), Serra (2011), Hummel (2013). In Appendix E I show that results are robust to the use of more general functional forms.

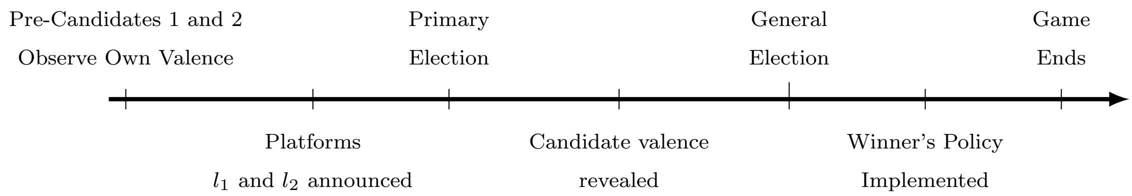
[15] The insights of the model are robust to considering a general valence level for the incumbent.

some probability, either before the primary election or after the general election[16]. All candidates are purely office motivated and risk neutral. Using a convenient normalization, their utility is 1 if they win the general election and zero otherwise.

When voting at the primary election stage, voters are forward-looking: that is to say, they trade-off their policy preference for each pre-candidate with the probability that each of them wins the general election. As a result, voting in the primary election is akin to choosing between lotteries respectively delivering each L party pre-candidate's policy platform in case of victory in the general election and the R party policy platform r in case of defeat. Primary voters vote for the pre-candidate associated with the highest expected utility. I assume that if indifferent between two candidates, a voter breaks the tie in favor of the candidate with the higher valence.

The timing of the game, as summarized by Figure 2, is as follows: first, two pre-candidates, labeled by 1 and 2, are randomly drawn to run for the nomination in the L party. Having observed their own valence, the two pre-candidates simultaneously announce their policy platforms, which remain binding throughout the electoral process. Primary election voters observe the policies of the two primary candidates, update their beliefs on the pre-candidates' types and cast their votes in the primary election. The winner of the primary becomes the L party candidate in the general election and runs against the R party incumbent. Before the general election, valence becomes public knowledge. All voters then cast their votes in the general election: the policy platform of the winner is implemented and payoffs are realized.

Figure 2: Game Timeline



Before proceeding I introduce some of the notation that will be used throughout the paper. Denote by l the policy platform chosen by an L party general election candidate. Given this policy-platform and type $\theta \in \{A, D\}$, the L party candidate has a probability

[16]Snyder and Ting (2011) take a similar approach, assuming that valence can be revealed with some probability at each stage of the electoral campaign. In most models of primaries, valence is either known before primary elections, as in Hummel (2013) and Casas (2019), or revealed by the primary election campaign, as in Adams and Merrill III (2008) and Serra (2011); in Kartik and McAfee (2007), valence is private information and never revealed, and in Boleslavsky and Cotton (2015) it is unknown by everyone and revealed by the general election campaign.

$P_\theta(l)$ of winning the general election. The expected utility a voter with bliss point x derives from an L party candidate of type θ running against incumbent r is thus:

$$\mathbb{E}_\mu[u_x(y, v(y))|l, \theta] = P_\theta(l)u_x(l, \theta) + [1 - P_\theta(l)]u_x(r, v)$$

where $y \in \{l, r\}$ is the policy platform of the election winner and $v(y)$ her valence. The expectation is taken with respect to the realization of μ, i.e. the location of the general election median voter. In order to make notation lighter, I introduce the function $W_x(l, \theta)$ defined as follows:

$$W_x(l, \theta) \equiv \mathbb{E}_\mu[u_x(y, v_y)|l, \theta] \tag{3}$$

When voting in the primary election, however, voters cannot observe pre-candidates' types. As a result, they form beliefs $\nu(l)$ based on the policy l chosen by a pre-candidate. As usual in Bayesian games, beliefs follow Bayes rule when possible. With this in mind, the expected utility from a pre-candidate becomes $\mathbb{E}_\theta[W_x(l, \theta)] = \mathbb{E}_\theta[\mathbb{E}_\mu[u_x(y, v_y)|l, \theta]]$, that is:

$$\mathbb{E}_\theta[W_x(l, \theta)] = \nu(l)\mathbb{E}_\mu[u_x(y, v_y)|l, A]] + (1 - \nu(l))\mathbb{E}_\mu[u_x(y, v_y)|l, D]] \tag{4}$$

and therefore when comparing two pre-candidates l_1 and l_2, a primary voter located at x votes for l_1 whenever:

$$\mathbb{E}_{\theta_1}[W_x(l_1, \theta_1)] \geq \mathbb{E}_{\theta_2}[W_x(l_2, \theta_2)] \tag{5}$$

Given the behavior of primary election voters, $\mathbb{E}_{\theta_2}\left[P^{pr}(l_1, l_2(\theta_2))\right]$ represents the probability of winning the primary election for pre-candidate 1 choosing policy-platform l_1 against pre-candidate 2, taking expectations over the type of pre-candidate 2 and the resulting policy l_2. Conditionally on winning the primary, the probability of winning the general election is, as introduced above, $P_{\theta_1}(l_1)$, keeping in mind that valence becomes observable before the general election. The probability of winning office when choosing policy-platform l_1 is therefore:

$$\mathbb{E}_{\theta_2}\left[P^{pr}(l_1, l_2(\theta_2))\right] P_{\theta_1}(l_1) \tag{6}$$

The equilibrium concept I use in this paper is Perfect Bayesian Equilibrium (PBE). I focus on symmetric pure strategy equilibria, where by symmetry I mean that the choices of a pre-candidate only depend on her type and not on her label as pre-candidate 1 or pre-candidate 2. The following definitions summarize the structure of the game and the concept of Perfect Bayesian Equilibrium.

Definition 1. *The players are the following:* 2 *pre-candidates in the L party, indexed by* 1 *and* 2*, and a set of L party primary voters*[17].

The strategy S_i of each pre-candidate $i \in \{1,2\}$ is a mapping between their type and a policy platform on the real line, conditional on all the information which is common knowledge[18]*:*

$$S_i : \theta_i \to \mathbb{R}$$

A pure strategy of a primary voter with bliss-point x, denoted by B_x^{pr}, is a mapping between the voter's bliss point and the policy platforms chosen by pre-candidates on one side and a vote cast for either pre-candidate 1 *or* 2 *on the other side:*

$$B_x^{pr} : x \times \{l_1, l_2\} \to \{1,2\}$$

A pure strategy of a general election voter B_x is a mapping between the policy bliss-point and the platforms and valence levels of candidates on one side and a vote cast for the candidate of party L or that of party R:

$$B_x : x \times \{l, r, v_l, v_r\} \to \{L, R\}$$

Definition 2. *A pure strategy PBE of the primary election game consists of the following elements:*

1. *A belief function $\nu(l) = Pr(A|l)$, which associates to each policy choice of pre-candidates from the L party a probability of being of the valent type A. Beliefs are consistent with Bayes rule on the equilibrium path.*

2. *Given S_1, S_2 and beliefs $\nu(l)$, each primary voter chooses B_x^{pr} in order to maximize their expected utility* (4)*. General election voters choose B_x to maximize* (1)*.*

3. *Given B_x^{pr}, B_x, $\nu(\cdot)$ and S_{-i}, each pre-candidate $i \in \{1,2\}$ chooses S_i in order to maximize expected utility as given by condition* (6)*.*

Clearly, this being a signaling game, there is the issue of multiplicity of equilibria: in Section 5 I rank equilibria in terms of welfare of primary election voters and show that

[17] The R party incumbent is a passive player and takes no action.

[18] Including the policy platform r and the valence v_r of the R party incumbent, the distributions of pre-candidate types, the L party median voter location m, the distribution of the general election median voter's bliss point μ and the utility functions of all players.

only two focal equilibria can survive a commonly used selection criterion for signaling games, the D1 criterion introduced by Cho and Kreps (1987).

4 Results

I start the analysis from the general election in which the party R incumbent candidate faces the winner of party L primary. The policy platforms of the two candidates are l and r respectively; their valence levels v_l and v_r are publicly observable. Following previous discussions, $v_r = v$ and $v_l \in \{0, v\}$. The outcome of the general electiondepends on the comparison between the bliss-point of the median voter μ and that of the voter indifferent between the policy platforms proposed by the two candidates, which I denote by z. Notice that, given the uniform distribution of μ, for all $z \in [-b, b]$ it holds that $Pr(\mu \leq z) = \frac{z+b}{2b}$. Therefore, z determines the probability $P_\theta(l)$ of the L party candidate winning the general election, conditional on her type θ.

Lemma 1. *Suppose $l \leq r$. The indifferent voter z takes the following location depending on l, r and θ:*

$$z = \begin{cases} -\infty, & \text{if } v_r > v_l \text{ and } v_r \geq r - l \\ \frac{l+r+v_l-v_r}{2}, & \text{if } \max\{v_r, v_l\} < r - l \\ +\infty, & \text{if } v_l > v_r \text{ and } v_l \geq r - l \end{cases} \tag{7}$$

Given z, the probability of candidate l of winning the election is the following:

$$P_\theta(l) = \begin{cases} 0, & \text{if } z < -b \\ \frac{z+b}{2b}, & \text{if } z \in [-b, b] \\ 1, & \text{if } z > b \end{cases} \tag{8}$$

Given the possibility of corner solutions highlighted in Lemma 1, I make the following assumption:

Assumption 2. *The following restrictions hold: i) $0 \leq r < 3b$ and ii) $v < b$.*

Assumption 2 makes sure that the election between the right party candidate at r and a left party candidate at $-b$ does not lead to certain victory for either of the two candidates. Moreover, notice that $v < b$ has a natural interpretation: the voters of the L party always prefer a non valent candidate at $-b$ to a valent one at 0. In other words, L party voters never vote for the R party incumbent.

The following lemma establishes that absent signaling concerns, all voters with bliss-point smaller than $-b$, and hence all L party primary election voters, would choose $-b$ as the optimal policy platform.

Lemma 2. *Fix the probability that a candidate is valent to any value in* $[0,1]$. *The policy platform maximizing expected utility* (4) *for voters with bliss-points* $x \leq -b$ *is* $-b$.

The fact that the optimal candidate location does not depend on any parameter other than b is a consequence of the uniform distribution in combination with linear policy preferences. In particular, r and v end up having no effect on the optimal location of the candidate, since the effect they have on the probability for l of winning the election is offset by the opposite effect on the utility of the voter conditional on l winning the election. However, as I show in Appendix D, the fact that all sufficiently extreme voters have the same optimal location for a primary candidate is a general feature of a model with linear utility. As a consequence of Lemma 2, in a game where valence is private information of candidates at the time of platform choice but in which it becomes publicly observable before the primary election, rather than before the general election as I assume in this paper, the equilibrium outcome of the game would be that all candidates choose platform $-b$. Therefore, $-b$ can also be interpreted as the benchmark platform of a primary election held under full information over candidates' valence. The next result establishes that the median voter theorem holds also for the primary election, that is the L party median voter is decisive when comparing two pre-candidates (with potentially different expected valence) in the primary election. Notice that despite the single-peakedness of the utility function $u_x(l, \theta)$, this result is not obvious, because of the possibility of "strategic" voting by the right tail of the L primary electorate to favor the R party candidate in the general election. Therefore, Assumption 1 is a relatively mild restriction on the preferences of primary voters that also works as sufficient condition for the median voter theorem to hold[19].

Proposition 1. *Under Assumption 1, the median voter is always decisive in the primary election. That is, pre-candidate* 1 *wins the primary election if* $\mathbb{E}_{\theta_1}[W_m(l_1, \theta_1)] >$

[19]Without restrictions on the bliss-point of primary voters, one can construct examples in which voters in the center of the primary electorate prefer one pre-candidate, whereas both voters in the left and right tail of the primary electorate support the other one. Whereas the former choose such pre-candidate for ideological affinity, the latter support a candidate ideologically far from their bliss-point in order to facilitate the victory of the R party incumbent. Although there is some anecdotal evidence of this kind of behavior, it is by no means prevalent, and it is especially unlikely in closed primary elections, that is the type of primaries most related to my analysis. Therefore, also in the interest of tractability, I rule out this possibility by assuming L party primary voters are sufficiently to the left in the ideological spectrum.

$\mathbb{E}_{\theta_2}[W_m(l_2, \theta_2)]$ *and pre-candidate* 2 *wins if* $\mathbb{E}_{\theta_1}[W_m(l_1, \theta_1)] < \mathbb{E}_{\theta_2}[W_m(l_2, \theta_2)]$. *If instead* $\mathbb{E}_{\theta_1}[W_m(l_1, \theta_1)] = \mathbb{E}_{\theta_2}[W_m(l_2, \theta_2)]$, *the candidate with the highest valence wins the primary, or each wins with probability* 1/2 *if they have the same valence*[20].

Before moving to the analysis of the primary election in the L party, I provide another preliminary result, which establishes a single crossing property that I will use repeatedly in the solution of the model[21]:

Lemma 3. *For any* $l_2 > l_1$ *such that* $P_A(l_1) > 0$ *and* $P_D(l_2) > 0$,

$$\frac{P_D(l_1)}{P_D(l_2)} < \frac{P_A(l_1)}{P_A(l_2)} \tag{9}$$

The interpretation of the result in Lemma 3 is that moving from platform l_2 to the more extreme platform l_1, the probability of winning the general election for non-valent pre-candidates decreases proportionally more than for valent pre-candidates. This property plays a key role for the existence of extremist separating equilibria.

4.1 Pooling Equilibria

In a pooling equilibrium, pre-candidates of both types choose the same platform, which I denote by l^{pool}. Since pre-candidates are ex-ante identical, each of them wins the primary election with probability 1/2. In order to minimize the extent of profitable deviations available to pre-candidates and thus describe the set of all possible pooling equilibria, I fix out-of-equilibrium beliefs[22] at $\nu(l) = 0$ for all $l \neq l^{pool}$. The existence of a pooling equilibrium requires that, for both types $\theta \in \{D, A\}$ and for all policy platforms l, the following holds:

$$\frac{1}{2}P_\theta(l^{pool}) \geq P^{pr}(l, l^{pool})P_\theta(l) \tag{10}$$

Notice that, by Lemma 2, primary voters would choose to locate a pooling candidate at $l = -b$. As a result, for l^{pool} far enough from $-b$, the policy distortion is so large that primary voters would prefer a pre-candidate who is certainly non-valent but who chooses platform $-b$ rather than a pooling candidate at l^{pool}. Given that this holds for values of l^{pool} both too much to the left or too much to the right, there is an interval $[\underline{l}^{pool}, \bar{l}^{pool}]$ of

[20]The assumption that if the median voter is indifferent, the candidate with the highest valence wins the primary election is just chosen to make the characterization of equilibria more (esthetically) convenient by involving closed sets. Relaxing this assumption would pose no problem for the solution of the model.

[21]In Appendix D and E I show that single crossing continues to hold under non-uniform distributions of the general election median voter location as well as under non-linear policy preferences.

[22]See Lemma A.1 in Appendix A for a formal statement of why this can be done without loss of generality.

policy platforms that can be part of a pooling equilibrium.

$$\mathbb{E}_\theta[W_m(\underline{l}^{pool}, \theta)] = \mathbb{E}_\theta[W_m(\bar{l}^{pool}, \theta)] = W_m(-b, D) \tag{11}$$

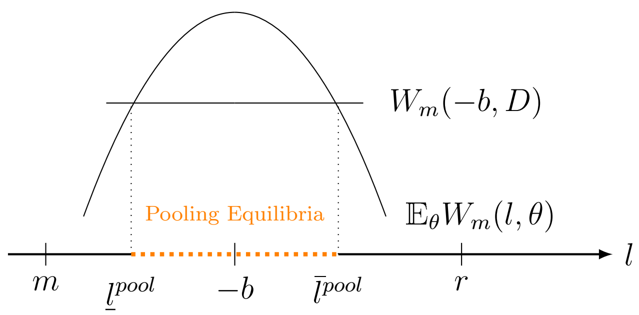
The set of existing pooling equilibria is never empty and always contains $-b$, as displayed in Figure 3.

Proposition 2. *Policy platform l can be part of a pooling equilibrium if and only if, given beliefs $\nu(l) = \alpha$ and $\nu(l') = 0$ for all other $l' \in \mathbb{R}$,*

$$\mathbb{E}_\theta[W_m(l, \theta)] \geq W_m(-b, D), \tag{12}$$

where $\mathbb{E}_\theta[W_m(l, \theta)] = \alpha W_m(l, A) + (1 - \alpha) W_m(l, D)$. This defines an interval $[\underline{l}^{pool}, \bar{l}^{pool}]$ of possible pooling equilibria, with $\underline{l}^{pool} < -b < \bar{l}^{pool}$.

Figure 3: Interval of Pooling Equilibria



In the example depicted here, $\underline{l}^{pool} > m$. The function $\mathbb{E}_\theta[W_m(l, \theta)]$ would display a kink at $l = m$.

4.2 Separating Equilibria

I now discuss the existence conditions for separating equilibria. In a pure strategy separating equilibrium, valent pre-candidates choose policy l_A whereas non-valent ones choose policy l_D, so that $\nu(l_A) = 1$ and $\nu(l_D) = 0$ constitute on-path beliefs[23]. In order for no pre-candidate to have a profitable deviation, the following condition must be satisfied for both types $\theta \in \{A, D\}$ and for all possible deviations $l' \in \mathbb{R}$.

$$\mathbb{E}_{\theta_2}\left[P^{pr}(l_\theta, l_2(\theta_2))\right] P_\theta(l_\theta) \geq \mathbb{E}_{\theta_2}\left[P^{pr}(l', l_2(\theta_2))\right] P_\theta(l') \tag{13}$$

[23] Analogously to the case of pooling equilibria, in order to describe the whole set of possible separating equilibria, I fix, without loss of generality, $\nu(l) = 0$ for all $l \neq l_A$.

To put more content into condition (13), the first thing to notice is that in any separating equilibrium, a valent pre-candidate can never lose the primary election against a non-valent one, but can only lose in a tie-break with another valent candidate. That is to say, $P^{pr}(l_D, l_A) = 0$, $P^{pr}(l_A, l_D) = 1$ and $P^{pr}(l_A, l_A) = P^{pr}(l_D, l_D) = 1/2$. This is a consequence of the single-crossing property derived in Lemma 3 and it rules out potential perverse separating equilibria in which valent candidates choose policies that are more desirable for the general election and primary election voters elect non-valent candidates if they have the choice. The expected probability for a pre-candidate of each type of winning the primary election can thus be rewritten as:

$$\mathbb{E}_{\theta_2}\left[P^{pr}(l_D, l_2(\theta_2))\right] = \frac{1-\alpha}{2} \tag{14}$$

and

$$\mathbb{E}_{\theta_2}\left[P^{pr}(l_A, l_2(\theta_2))\right] = 1 - \frac{\alpha}{2} \tag{15}$$

In order for this to be consistent with the utility maximizing behavior of a PBE, the platforms chosen by pre-candidates must be such that valent pre-candidates are preferred to non-valent ones by a majority of primary voters, that is:

$$W_m(l_A, A) \geq W_m(l_D, D) \tag{16}$$

Moreover, thanks to Lemma 2 it can also be established that in any pure strategy separating equilibrium, non-valent pre-candidates have to choose $l_D = -b$. Since l_D is associated with the worst possible belief $\nu(l_D) = 0$, l_D must correspond to the first-best policy platform $-b$.

Lemma 4. *In any separating equilibrium, $l_D = -b$.*

Having pinned down l_D, l_A must lie at the intersection of the following three sets: I denote by $\mathcal{P}^A_{-b}$ the set containing the values of l_A satisfying condition (16), given $l_D = -b$; analogously, the set $\mathcal{IC}^D_{-b}$ collects the values of l_A such that condition (13) is satisfied for $\theta = D$; finally, the set $\mathcal{IC}^A_{-b}$ contains the values of l_A such that condition (13) is satisfied for $\theta = A$[24].

Consider $\mathcal{IC}^D_{-b}$ first. Evaluating (13) for $l' = l_A$, $\theta = D$ and $l_D = -b$ yields:

$$\frac{P_D(l_A)}{P_D(-b)} \leq a \tag{17}$$

[24]Notice that, concerning condition (13), it is enough to check for deviations from l_D to l_A for types $\theta = D$ and from l_A to l_D for $\theta = A$ types. This follows the fact that $\nu(l) = 0$ for all $l \neq l_A$, such that any pre-candidate deviating to any policy $l' \notin \{l_A, l_D\}$ would receive a payoff of zero.

where I introduce parameter $a = \frac{1-\alpha}{2-\alpha}$ to denote the ratio between the expected equilibrium probability of winning the primary elections for a non-valent versus a valent pre-candidate. With respect to $\mathcal{IC}^A_{-b}$, an analogous procedure allows to rewrite (13) as:

$$\frac{P_A(l_A)}{P_A(-b)} \geq a \tag{18}$$

The logic behind conditions (17) and (18) is that the separating policy l_A needs to be sufficiently unattractive for non-valent pre-candidates but sufficiently attractive for valent ones. To sum up, we now have three conditions that l_A needs to satisfy in order for a separating equilibrium to exist. This is formalized by the following proposition.

Proposition 3. *A pair* (l_D, l_A) *can be part of a separating equilibrium as long as* $l_D = -b$ *and* $l_A \in \mathcal{IC}^D_{-b} \cap \mathcal{IC}^A_{-b} \cap \mathcal{P}^A_{-b}$*, which describes the points satisfying* (16)*,* (17) *and* (18)*. If* $\mathcal{IC}^D_{-b} \cap \mathcal{IC}^A_{-b} \cap \mathcal{P}^A_{-b} = \emptyset$*, no separating equilibrium exists.*

Having derived the general conditions for a separating equilibrium to exist, the next step is to characterize which policy platforms can be part of a separating equilibrium. Solving condition (17) as an equality yields an upper bound on l_A, located to the left of $-b$, which I denote as l^e_A and which takes the following form:

$$l^e_A = -b - (1-a)(b + r - v) \tag{19}$$

Policy platforms below this point are incentive compatible with respect to non-valent pre-candidates deviating to mimick valent ones. The intuition is that, given the type of pre-candidate, when the policy l_A is sufficiently far to the left, the probability of winning the general election with policy-platform l_A is low compared to that of winning it with platform $-b$. This makes non-valent pre-candidates prefer to reveal themselves as such. If on one hand this makes them win the primary election with a lower probability than if they were to mimick valent types, on the other hand it puts them in a better position to win the general election conditional on winning the primary. This is the key intuition for all the separating equilibria of the model.

However, notice that policies which are sufficiently to the left of $-b$ are not the only policies that allow valent pre-candidates to signal their type. Following Lemma 1, $P_D(l) = 0$ for all $l \in [r-v, r+v]$: in other words, a non-valent pre-candidate choosing a platform too close to the incumbent is bound to lose the election. The reason is that when the platforms chosen by candidates are sufficiently similar, all voters end up choosing the candidate with the highest valence. As a result, all points in $[r-v, r+v]$ satisfy (17) and are hence in $\mathcal{IC}^D_{-b}$.

Having established which policy platforms satisfy incentive compatibility for $\theta = D$ types (i.e. belong to the set $\mathcal{IC}^D_{-b}$), I now consider $\mathcal{IC}^A_{-b}$. Concerning the policy platforms to the left of l^e_A, the single-crossing property derived by Lemma 3 assures that $l^e_A \in \mathcal{IC}^A_{-b}$. Therefore, given $l_A = l^e_A$, valent candidates $\theta = A$ do not have the incentive to deviate to $l_D = -b$. The intuition is that thanks to single-crossing, when non-valent candidates are indifferent between revealing themselves as $\theta = D$ and mimicking valent candidates, valent candidates strictly prefer to reveal themselves as $\theta = A$ by choosing l_A.

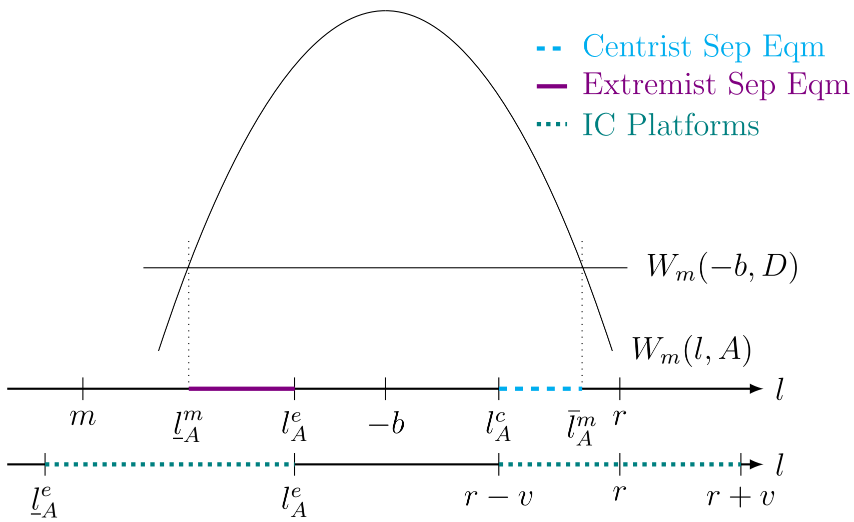
Moving further to the left of l^e_A, the policy chosen by the valent pre-candidate becomes increasingly disadvantageous for the general election. Hence, at some point, valent candidates have the incentive to deviate and choose the policy platform assigned to non-valent pre-candidates. This defines a lower-bound $\underline{l}^e_A$ for $\mathcal{IC}^D_{-b} \cap \mathcal{IC}^A_{-b}$. Concerning the policies in $\mathcal{IC}^D_{-b}$ that lie in $[r - v, r + v]$, notice that all those satisfying $l_A < r$ are also part of $\mathcal{IC}^A_{-b}$. The reason is that on top of signaling valence, they also grant valent types a larger probability of winning the election than $l_D = -b$.

The final condition to be satisfied by a separating equilibrium is (16), that is the fact that the platform l_A has to belong to $\mathcal{P}^A_{-b}$. Similarly to what happened in the characterization of the pooling equilibria, the set $\mathcal{P}^A_{-b}$ defines an interval of policy platforms that deliver the median voter in the primary election a greater expected utility than platform $-b$ being chosen by a non-valent candidate. Denote this interval as $[\underline{l}^m_A, \bar{l}^m_A]$. If $\underline{l}^m_A > l^e_A$, none of the policies in $[\underline{l}^e_A, l^e_A]$ can be part of a separating equilibrium. The reason is that even the most moderate of such policies, i.e. l^e_A, is too extreme for the median primary voter to prefer it to a non-valent candidate choosing $-b$. If instead $\underline{l}^m_A \leq l^e_A$, then platforms in $[\min\{\underline{l}^e_A, \underline{l}^m_A\}, l^e_A]$ can be part of a separating equilibrium. Similarly with regards to policies in $[r - v, r)$, if $\bar{l}^m_A < l^c_A$ none of those policies can be part of a separating equilibrium, whereas if $\bar{l}^m_A \geq l^c_A$ then policies in $[l^c_A, \bar{l}^m_A]$ can be part of a separating equilibrium. Again, the intuition is that such separating equilibria exist as long as the separating platform closest to $-b$, that is l^c_A, is preferred to platform $-b$ chosen by a non-valent candidate. Notice that $\bar{l}^m_A < r$, so that policies in $[r, r + v]$ can never be part of a separating equilibrium.

Proposition 4. *If a separating equilibrium exists, l_A is in one of the two intervals $[\min\{\underline{l}^e_A, \underline{l}^m_A\}, l^e_A] \subset (-\infty, -b)$ and $[l^c_A, \bar{l}^m_A]$, with $l^c_A = r - v$ and $\bar{l}^m_A < r$.*

As Figure 4 displays graphically, there can exist two types of separating equilibria, that differ in terms of the policy chosen by valent pre-candidates. If $l_A \in [\min\{\underline{l}^e_A, \underline{l}^m_A\}, l^e_A]$, valent pre-candidates signal their valence by choosing a more extreme policy than their non-valent opponents, and therefore I call these equilibria *extremist* separating equilibria. If instead $l_A \in [r - v, \bar{l}_A^m]$, valent pre-candidates choose a policy closer to the incumbent, and also closer to the center of the ideological space: for this reason I call these *centrist* separating equilibria[25].

Figure 4: Extremist and Centrist Separating Equilibria



In the example depicted here, $\underline{l}_A^e < \underline{l}_A^m$. The opposite might however be true. Moreover, notice that the function $W_m(l, A)]$ would display a kink at $l = m$.

As Proposition 3 makes clear, a separating equilibrium, unlike a pooling equilibrium, is not guaranteed to exist for all parameter combinations. The next corollary explicitly shows necessary and sufficient conditions for at least one centrist and extremist separating equilibrium to exist. These conditions are nothing other than the existence conditions of the centrist equilibrium with $l_A = l_A^c$ and the extremist one with $l_A = l_A^e$. As a matter of fact, if any centrist (respectively extremist) separating equilibrium exists, the centrist (respectively extremist) at l_A^c (respectively l_A^e) exists. As we will see in the next section, these equilibria are also focal in that, together with the pooling equilibrium at $-b$, they make up the set of equilibria that can be welfare dominant for primary election voters.

Corollary 1. *The set of existing centrist (and respectively extremist) separating equilibria is not empty if and only if condition* (16) *is satisfied at* $l_A = l_A^c$ *(and at* $l_A = l_A^e$ *respectively).*

Notice that condition (16) applied to the centrist separating equilibrium does not

[25]In the comparative statics paragraph I provide a more detailed description of the meaning of centrist.

depend on either m or α, but only on the value of v compared to b and r, which reads:

$$\frac{(b+r-v)^2}{4b} - v\min\left\{1, \frac{2(b+r)-v}{4b}\right\} \leq 0 \tag{20}$$

Following this condition, the centrist separating equilibrium exists as long as valence v is sufficiently large or b and r are sufficiently small[26]. In other words, the centrist separating equilibrium requires high valence, low uncertainty over the bliss-point of the median voter b and a moderate platform of the incumbent r. The condition for the existence of extremist separating equilibria, on the other hand, also depends on α and m (on the latter only as long as $m > l_A^e$). In particular, evaluating (16) at l_A^e delivers the following condition:

$$m \leq r - |l_A^e - m| - \frac{(b+r-v)^2}{v + a(b+r-v)} \tag{21}$$

If $m \geq l_A^e$, condition (21) translates into an upper bound on m below which the extremist separating equilibrium at l_A^e exists. If instead $m < l_A^e$, then condition (21) can be rewritten as a parametric condition similar to condition (20). Just like the centrist separating equilibrium, the extremist one also requires high valence v, low uncertainty b and a moderate incumbent platform r. Besides that, the extremist separating equilibrium requires a sufficiently low value of α, which can be strictly lower than 1: the reason is that if α is too large, the platform allowing the valent candidate to separate might be too extreme for the median voter to prefer it over platform $l_D = -b$.

5 Comparative Statics and Equilibrium Selection

As it has become clear in Section 4, there are many possible equilibrium outcomes, as is always the case in signaling games. In particular, there is an interval of pooling equilibria and two intervals of possible separating equilibria, extremist and centrist. However, each of these three intervals contains a focal equilibrium which, from the point of view of primary voters' welfare, dominates the others of the same kind.

The characterization of the welfare dominant equilibria of the game follows directly from Lemma 2: comparing two pre-candidates with the same probability of being valent, primary voters prefer the one closer to the first best platform $-b$. Comparing separating equilibria, the first thing to notice is that l_D is equal to b across all such equilibria

[26] The second term in condition (20) reflects the fact that in some circumstances, in particular if r is too much to the right, the valent candidate in a centrist separating equilibrium wins the general election with certainty.

(independently of whether centrist or extremist); in addition to that, the probability of selecting a valent pre-candidate is equal to $1-(1-\alpha)^2 = \alpha(2-\alpha)$ in all separating equilibria. The comparison between the expected utility of any two separating equilibria can thus be reduced to a comparison between the expected utility from the policy platforms l_A chosen by valent pre-candidates. Hence, Lemma 2 can be applied to conclude that the welfare maximizing extremist and centrist separating equilibria are respectively that with $l_A = l_A^e$ and that with $l_A^c = r - v$. An analogous reasoning allows to conclude that the welfare maximizing pooling equilibrium is such that $l^{pool} = -b$.

It is also important to notice that the pooling equilibrium at $l^{pool} = -b$ always exists and, given Corollary 1, the separating equilibria with $l_A \in \{l_A^e, l_A^c\}$ always exist unless no separating equilibrium of this kind exists. This means that the welfare dominant equilibrium of the game is always one of the three focal equilibria I described.

Proposition 5. *For all parameter combinations, the welfare dominant equilibrium for primary election voters is one of the following three: i) The pooling equilibrium with* $l^{pool} = -b$*, ii) The extremist separating equilibrium with* $l_A = l_A^e$ *defined by* (19) *and iii) The centrist separating equilibrium with* $l_A = l_A^c$.

5.1 Comparative Statics

Having identified the welfare dominant equilibria of the game, I can now perform some comparative statics on the platforms l_A^e and l_A^c. Let's start from the extremist separating equilibrium: keeping in mind that $l_A^e = -b - (1-a)(b + r - v)$, where $a = \frac{1-\alpha}{2-\alpha}$. The comparative statics are straightforward, but the most effective way to glean intuition is to remember that l_A^e solves condition (17), i.e. $\frac{P_D(l_A)}{P_D(-b)} \leq a$, as equality. Consider first the parameter α: this appears only on the right-hand side of (17): the larger α, the smaller the ratio a, i.e. the less likely non-valent pre-candidates are to win primaries compared to valent pre-candidates. To preserve incentive compatibility, l_A^e moves to the left. Parameters r, b and v instead enter the ratio on the left hand side of (17): an increase in r or b increases the probability of winning the general election with the extremist policy l_A^e proportionally more than with the non-valent policy $-b$. Therefore, incentive compatibility requires a more extreme l_A^e. The effect of an increase in v is the opposite, given the substitutability between r and v in (17). As a result, policy l_A^e is less extreme when valence is higher[27].

This has some interesting implications: first, the "extremism" of the incumbent's platform r has a polarizing effect on the platform choice of a valent challenger. In this

[27]Notice that the valence v entering condition (17) is the valence of the incumbent.

respect, my model uncovers a novel channel to explain how polarization in one party can have reinforcing effects on the polarization of the opposing party. The second implication has to do with valence: although the extremist separating equilibrium implies that valent pre-candidates choose more extreme platforms than non-valent ones, the positive correlation between valence and extremism is mitigated by the fact that the higher valence, the more moderate is platform l_A part of an extremist separating equilibrium.

A similar analysis can be carried out for the centrist equilibrium: in that case, the L party pre-candidate "tracks" the policy of the R party candidate. Hence, policy platform $l_A^c = r - v$ moves to the right as r increases but it moves to the left as v increases (whereas it is not affected by either b or α). Notice that $|l_A^c| < |l_A^e|$ hence the notion of centrist equilibrium, but there are situations where $l_A^c > 0$ and the centrist equilibrium exists[28].

Proposition 6. *The extreme separating equilibrium policy l_A^e moves to the left whenever: i) α increases ii) b increases iii) r increases or iv) v decreases. The centrist separating equilibrium policy l_A^c moves to the right as r increases and to the left as v increases.*

Proposition 6 tells us how the policies chosen in each equilibrium change when the parameters of the model change. However, changes in the parameters can also cause a switch in the welfare dominant equilibrium, either between separating and pooling, or between extremist separating and centrist separating. Consider first the comparison between the centrist and extremist separating equilibria. Intuitively, the more extreme primary voters are, the more likely they are to prefer the extremist separating equilibrium. In particular, since the probability of a valent candidate being elected is the same in both equilibria, the comparison between the two equilibria depends on the expected policy platform implemented given each equilibrium. For example, voters with a bliss point to the left of l_A^e prefer the separating equilibrium which delivers the most left-wing expected policy platform. This is the extremist equilibrium when the following condition is satisfied:

$$P_A(l_A^e)l_A^e + (1 - P_A(l_A^e))r \leq P_A(l_A^c)l_A^c + (1 - P_A(l_A^c))r \tag{22}$$

This condition is necessary for a majority of voters to prefer the extremist separating equilibrium, providing another justification for the term *extremist*. However, it is not sufficient, since the median primary election voter might have a bliss-point m to the right

[28]The fact that $|l_A^c| < |l_A^e|$ holds since the following inequality always holds under Assumption 2: $|r - v| < |-b - (1 - a)(b + r - v)|$. Moreover, without further parameter restrictions, it is possible for the valent L party pre-candidate in a centrist separating equilibrium to win the general election with certainty against the incumbent. This happens when $\frac{r-v+r}{2} > b$. Intuitively, if the incumbent is to the right of b and valence v is small enough, the centrist pre-candidate can be enough to the right to always win the general election.

of l_A^e. In that case, the median primary voter prefers the extremist separating equilibrium if:

$$m \leq \frac{l_A^c + l_A^e}{2} - \frac{v}{2}\left[\frac{P_A(l_A^c)}{P_A(l_A^e)} - 1\right] \tag{23}$$

Condition (23) can be interpreted as follows: the first term represents the mid-point between the policy-platform of the valent candidate in the centrist versus the extremist separating equilibrium. The second term, instead, represents the adjustment to take account for the different probabilities of winning the general election at l_A^c and l_A^e. The more likely the centrist candidate is to win the general election compared to the extremist one, the more the threshold moves to the left.

The following proposition summarizes these facts into a unique condition, which reduces to (22) for $m \leq l_A^e$ and to (23) if $m > l_A^e$.

Proposition 7. *A majority of primary election voters prefers the extremist to the centrist separating equilibrium if and only if:*

$$m \leq l_A^c - l_A^e - v\left(\frac{P_A(l_A^c)}{P_A(l_A^e)} - 1\right) - |l_A^e - m| \tag{24}$$

An interesting additional implication of Proposition 7 is that small changes in the ideology of primary election voters might have large effects on the outcome of primary elections by changing the way valent pre-candidates signal their valence in primary elections. This feature of the model can for example remind of the change in electoral narrative that has been seen in recent primary elections in the United States: several candidates have, in an increasingly successful manner, chosen radical platforms instead of more traditional centrist ones. My model can reconcile this observation with an environment in which voters' ideological preferences have remained more or less constant, as argues for example by Fiorina and Abrams (2008).

Independently of what type of separating equilibrium voters prefer for some given parameters, the ranking between pooling and separating equilibria follows the same pattern. As it can be seen in Figures 5a and 5b, a separating equilibrium is preferred for high values of valence v (with respect to b and r) and low values of the share of valent pre-candidates α. This is connected to the comparative statics described in Proposition 3: when r increases, the separating policy platform l_A (be it extremist or centrist) moves further away from the first-best value $-b$, making it less appealing. The opposite happens after an increase in v, which also makes it more valuable to select a valent pre-candidate. Finally, an increase in α both makes l_A^e more extreme (in case of an extremist separating equilibrium) and makes a pooling equilibrium more appealing, since the risk of drawing a bad candidate decreases: as a result, a separating equilibrium is more likely to be preferred when α is not too high.

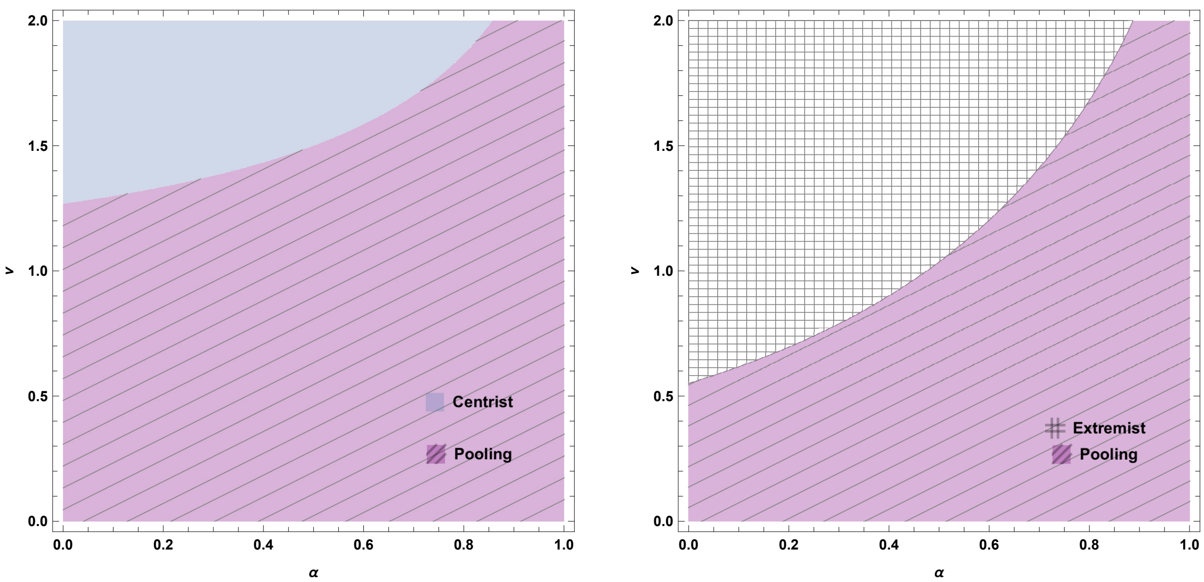
Proposition 8. *A separating equilibrium is preferred to the pooling when* v *is large enough (given* b *and* r*) and* α *is small enough. For large enough* r *the pooling equilibrium is preferred to both separating equilibria.*

Concerning the probability of winning the general election for the L party, the centrist separating equilibrium always increases it compared to the pooling equilibrium. However, this is not always the case for the extremist separating equilibrium. When such an equilibrium is played, the electoral prospects for the L party improve if the benefits of increased candidate valence are not offset by the costs of a more extreme platform. This happens when the following condition is satisfied:

$$\frac{v}{b+r} \geq \frac{1}{2-\alpha} \tag{25}$$

Finally, the ranking between separating and pooling equilibrium is also interesting because it can be interpreted as a way to compare nomination through primary elections and direct party nomination, as I will discuss more in depth in Section 6.

Figure 5: Welfare Dominant Equilibria



(a) Moderate Party Median Voter

Figure 5a shows the welfare dominant equilibrium of the game in the (α, v) space for the parameter values: $m = -2$, $b = 2$, $r = 1$

(b) Extreme Party Median Voter

Figure 5b shows the welfare dominant equilibrium of the game in the (α, v) space for the parameter values of Parameter values: $m = -5$, $b = 2$, $r = 1$

5.2 Equilibrium Selection

There are two main sources of equilibrium multiplicity in this game: one is the typical multiplicity of signaling games, given by the fact that unrestricted beliefs can lead to a multitude of equilibria. The other has to do with the existence of two different types of separating equilibrium, which often coexist for the same parameter combinations. Moreover, whereas primary election voters prefer one or the other type of equilibrium depending on the circumstances, valent pre-candidates, being purely office motivated, always prefer the centrist separating to the extremist separating equilibrium. This adds a further level of complexity.

In order to take care of the first source of equilibrium multiplicity, I apply a well-known equilibrium selection criterion for signaling games, the D1 criterion introduced by Cho and Kreps (1987). Restricting off-equilibrium beliefs, this eliminates all pooling equilibria as well as all separating equilibria other than the two focal ones with $l_A \in \{l_A^e, l_A^c\}$.

Proposition 9. *The only pure strategy equilibria that can survive the D1 criterion selection are the extremist separating equilibrium with $l_A = l_A^e$ and the centrist separating equilibrium with $l_A = l_A^c$. [one or the other] The D1 criterion always destroys all pooling equilibria, as well as all separating equilibria other the extremist separating equilibrium with $l_A = l_A^e$ and the centrist separating equilibrium with $l_A = l_A^c$.*

One downside of equilibrium selection through the D1 criterion is therefore that all pooling equilibria are destroyed, including the one at $-b$ that can be welfare dominant.

However, the D1 criterion has another downside: by restricting beliefs it gives additional leverage to valent pre-candidates, potentially allowing them to exploit their first mover advantage and achieve an electorally more competitive platforms, even when this goes against the interests of primary voters. Ultimately, this has to do with competition between valent pre-candidates not being perfect: when α is low, as a matter of fact, valent pre-candidates are almost monopolists in their superior valence and are thus not disciplined to choose the outcome preferred by primary voters.

To see this point more in detail, consider that under the D1 criterion, beliefs are restricted to $\nu(l) = 1$ for all $l \in [r - v, r + v]$. This gives valent pre-candidates the incentive to deviate to the policy that gives them the highest probability of winning the general election subject to still managing to win the primaries against a non-valent candidate. This policy is, by definition $\bar{l}_A^m$ solving $W_m(\bar{l}_A^m, A) = W_m(-b, D)$.

Therefore, a separating equilibrium (centrist or separating) exists only if the following condition is satisfied, for $l_A = l_A^c$ or $l_A = l_A^e$ respectively:

$$\frac{2-\alpha}{2} P_A(l_A) \geq (1-\alpha) P_A(\bar{l}_A^m) \tag{26}$$

As a result, as Proposition 10 makes clear, there are several instances in which the D1 criterion can fail to select the welfare dominant equilibrium: the first, already mentioned above, is the failure to select welfare dominant pooling equilibria. The second is the possibility of not selecting centrist or extremist separating equilibria that are welfare dominant: this happens when condition (26) is not satisfied for either l_A^c or l_A^e.

Finally, there is also the issue of uniquely selecting the welfare dominant equilibrium through the D1 criterion. If the welfare dominant equilibrium is the centrist one and condition (26) is satisfied, then the D1 criterion uniquely selects it. If instead the extremist separating equilibrium is welfare dominant and (26) is satisfied, it need not necessarily be selected as the unique pure strategy equilibrium of the game.

Proposition 10. *The D1 criterion selects the welfare dominant equilibrium only if this is separating. If the welfare dominant equilibrium is the centrist separating, then D1 selects it as long as condition* (26) *holds for $l_a = l_A^c$. If instead the welfare dominant equilibrium is the extremist separating, then D1 selects it if* (26) *holds for $l_a = l_A^e$, but it need not selected as the unique equilibrium of the game.*

In Figure 6 I present an example to show that the D1 criterion can fail to select the welfare dominant equilibrium for primary election voters. In particular, for the parameter combination presented in Figure 6, the extremist separating equilibrium is preferred to the centrist by a majority of primary voters. However, there is a large region of parameters (the blue region) for which the extremist separating equilibrium is destroyed by a deviation by valent pre-candidates to a centrist separating policy. Such a deviation is made possible by the D1 criterion. As a result, the extremist separating equilibrium exists only if either valence v is too low for the centrist separating equilibrium to exist (the checkered region) or when competition between valent candidates, measured by α, is high enough to prevent a centrist deviation (the region with parallel lines). However, for some of the parameters under which the extremist separating equilibrium is selected, the welfare dominant equilibrium would be the pooling. Finally, there are parameter values for which no pure strategy equilibrium is selected despite the extremist separating being welfare dominant.

An interesting consideration that follows this result is that despite the existence of an extremist separating equilibrium for a large set of parameter values, the scope for extremist policies to be part of an equilibrium is very much reduced by the interaction between the office motivation of candidates and the beliefs restrictions introduced by the D1 criterion. As a result of this, primary elections might in fact be likely to lead to "centrist" policies and benefit centrist voters and independents more than partisans with extreme bliss-points, even if the latter constitute a majority of the primary electorate.

There might of course be other forces acting as a counterweight to the combination of office motivation and D1-restricted beliefs: for example, dynamic considerations could push candidates to choose the equilibrium preferred by primary voters, in order to prevent a future primary challenge were they to choose the centrist policy. Similarly, candidates' choices might be regulated by social norms, or the support of possible platforms might be restricted by feasibility constraints due to the interaction with the party base or party interest groups necessary to create a party platform.

Figure 6: Equilibrium Selected by D1 criterion: Extremist Primary Median Voter

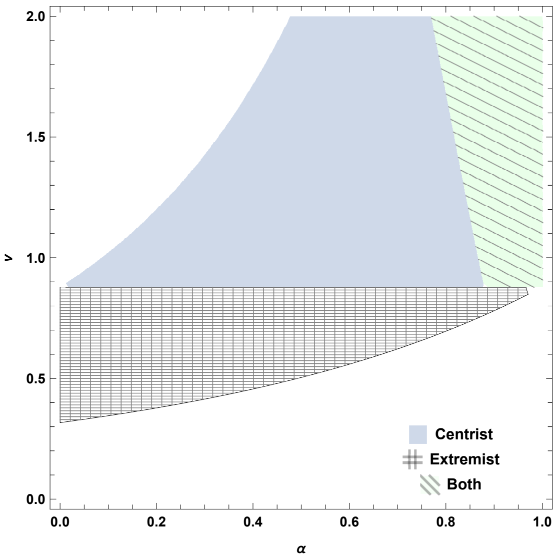


Figure 6 shows, in the (α, v) space, the equilibrium selected by the D1 criterion for parameter values: $m = -5$, $b = 2$, $r = 1$

6 Institutional Analysis

As I anticipated in the previous section, the ranking between separating and pooling equilibrium can be interpreted more broadly as a ranking between primary elections and direct party nomination. Notice that equating the pooling equilibrium with direct candidate nomination by party elites is equivalent to making the two following assumptions: i) Party elites draw their candidates from the same pool as the primary election and cannot (or do not want to) select valent candidates and ii) Party elites choose $-b$ as the policy platform under direct party nomination[29].

The equilibrium selection through the D1 criterion introduces a further twist on the institutional interpretation of the model: comparing Figure 5b and Figure 6, we can see that there are parameter values for which the welfare dominant equilibrium is the extremist separating, but where the unique equilibrium selected by the D1 criterion is the centrist separating. As a result, there are parameter values for which direct party nomination is preferred in order to achieve *the lesser evil*, that is the pooling equilibrium instead of the centrist separating.

Corollary 2. *Consider the game with equilibrium selection through the D1 criterion as in Proposition 9 and Proposition 10. If, as per Proposition 8, the welfare dominant equilibrium is the pooling equilibrium, then direct party nomination is preferred to primaries. If the welfare dominant equilibrium is the centrist separating, then primaries are preferred to direct nomination. If the welfare dominant equilibrium is the extremist separating, the ranking is ambiguous.*

To sum up, my model suggests that primary elections should (up to some further caveats due to equilibrium selection) be preferred to direct party nomination under the circumstances described in Proposition 8. These are the following: when direct selection by party elites is unlikely to result in a valent pre-candidate (low α); when valence is an important dimension for elections (i.e. parties that do not solely revolve around a specific ideology); when the incumbent's policy platform is not too extreme. Interestingly, my model does not suggest that primaries can only benefit parties with relatively extreme voters. In fact, the only situation in which primaries are guaranteed to be preferred to direct party nomination is when the welfare dominant equilibrium is the centrist separating, which requires sufficiently moderate primary voters.

This result suggests that an increased difficulty of identifying valent candidates as well as an increase in the importance of valence might have led parties to hold primaries. Such

[29]Concerning i), this assumption is a reasonable benchmark as it takes a neutral stance with respect to the potentially conflicting effects of primary elections on the probability of each candidate of being valent. In other words, it allows to single out the higher number of pre-candidates entering the race as the only source of increased selection of valent candidates through primaries. Concerning ii) this assumption is akin to abstracting from the representativeness of the party elite with respect to the base. This might of course be a relevant issue, but introducing them would only provide another reason to prefer primary elections. In Appendix C I briefly consider the "mirror" scenario in which both the assumptions I just discussed are relaxed. In particular, I focus on an environment in which nomination by party elites yields the same selection of valent candidates of a separating equilibrium (that is, party elites are no worse than primaries in selecting valent candidates), but where party elites would choose their own preferred policy platform.

forces seem to have indeed played a relevant role in the introduction of primaries in the United States at the beginning of the twentieth century. Urbanization and immigration deeply affected social cleavages, which probably made having valent politicians vital in order to keep voters together. Moreover, the same social changes mentioned above are also likely to have broadened the pool of possible politicians, making it harder for party insiders to judge the quality of candidates. The transition from a society in which most party voters would personally know candidates to a much more complex and changing environment is described by Ware (2002) in his book on the introduction of primary elections. Interestingly, Ware (2002) supports the view of primaries being not the result of a conflict between parties and voters, but the consequence of an evolution of society that made a renewal of the nomination system necessary for the very interests of parties.

With the due reverse causality caveats, this result also seems to be in line with the general observation that primaries are used in the United States, a country in which politics is more personalistic and candidate-centered, with traditionally a larger number of non-professional politicians entering politics from other sectors and where parties are considered to be less ideological than for example in Europe, where primaries are much less used.

7 Conclusion

This paper provides a theory of electoral competition in primary and general elections, focusing on an environment in which the valence of primary election candidates is private information.

In this setup, I find that office motivated candidates can use their choice of policy-platform to signal their valence. Interestingly, this can be done using two opposing strategies, that I call extremist and centrist. The former consists in choosing a sufficiently extreme policy platform: it might therefore be described as signaling strength by refusing to make policy compromises. The latter consists instead in "tackling" the opposing party candidate by choosing a platform that is close enough to his. The common feature of both these strategies is to involve platforms that are not worthwhile for non-valent pre-candidates: the extremist platform is too extreme for a non-valent candidate to be electorally competitive in the general election, whereas the centrist platform provides a non-valent candidate with too little differentiation with respect to the opposing party. In the extensions I show that extremist and centrist separating equilibria also occur when both parties hold primaries. In order for signaling to take place, the model predicts that three conditions have to be met: i) high relevance of valence in the electoral process ii) low quality of the candidates' pool and iii) sufficiently centrist opposing party candidate.

This model makes several contributions: from a theoretical point of view, it increases our understanding of the forces affecting policy choice in an environment with primary elections. In particular, it finds that asymmetric information over candidates' valence can have a substantial impact on the types of policies chosen in primary elections. In doing so, this paper is to the best of my knowledge the first to study signaling by valent candidates in a spatial model of primary elections.

From an applied point of view, my model speaks to two sets of stylized facts: first of all, it is consistent with the two-sided evidence on the effect of primary elections on policy extremism and polarization. Therefore, concerning the debate on whether primary elections cause polarization, my model points to the fact that primaries per se need not be a cause of policy extremism: in fact, they might even foster policy moderation. However, the combination of primary elections, extreme primary voters and uncertainty over candidates' valence can in some circumstances lead to substantial policy extremism. In addition to that, my model can also reconcile the negative and positive correlation between valence and policy extremism found by different studies. In this respect, the finding that policy extremism can be a price to pay to allow for the selection of better candidates can be useful to evaluate the welfare effects of policy polarization.

Finally, my model also sheds light on the conditions under which party voters are likely to benefit from primaries. Highlighting the importance of valence observability for candidate selection, it can for example provide an interesting perspective on the forces that have led to the introduction of primaries in the United States.

In future work, the setup developed in this paper could be used to answer additional questions: one avenue would be to include other possible ways of signaling valence and study questions related for example to campaign finance regulation; other possibilities would be to study the endogenous revelation of information on candidates or a setup with repeated elections.

Acknowledgements

I would like to thank the editor Marco Battaglini, an anonymous associate editor and three anonymous referees for their excellent comments and support throughout the revision process. As this project was part my PhD thesis, I am grateful to my advisors Andrea Mattozzi and David Levine. I would also like to thank Johanna Luise Reuter, Christopher Li, Elia Sartori, as well as audiences at the European University Institute and the 2016 EEA-ESEM Meeting in Geneva, for stimulating questions and comments.

References

Adams, J. and S. Merrill III (2008). Candidate and party strategies in two-stage elections beginning with a primary. *American Journal of Political Science 52*(2), 344–359.

Agranov, M. (2016). Flip-Flopping, Primary Visibility, and the Selection of Candidates. *American Economic Journal: Microeconomics 2*(8), 61–85.

Ansolabehere, S., J. M. Snyder Jr, and C. Stewart III (2001). Candidate positioning in us house elections. *American Journal of Political Science*, 136–159.

Aragones, E. and T. R. Palfrey (2002). Mixed equilibrium in a downsian model with a favored candidate. *Journal of Economic Theory 103*(1), 131–161.

Bernhardt, D., O. Câmara, and F. Squintani (2011). Competence and ideology. *The Review of Economic Studies 78*(2), 487–522.

Boleslavsky, R. and C. Cotton (2015). Information and extremism in elections. *American Economic Journal: Microeconomics 7*(1), 165–207.

Brady, D., H. Han, and J. Pope (2007). Elections and Candidate Ideology : Out of Step with the Primary Electorate? *Legislative Studies Quarterly 32*(1), 79–105.

Bullock, W. and J. Clinton (2011). More a molehill than a mountain: The effects of the blanket primary on elected officials' behavior from california. *The Journal of Politics 73*(3), 915–930.

Burden, B. C. (2001). The polarizing effects of congressional primaries. *Congressional primaries and the politics of representation*, 95–115.

Burden, B. C. (2004). Candidate positioning in us congressional elections. *British Journal of Political Science 34*(2), 211–227.

Carrillo, J. D. and M. Castanheira (2008). Information and strategic political polarisation. *The Economic Journal 118*(530), 845–874.

Casas, A. (2019). Ideological extremism and primaries. *Economic Theory*, 1–32.

Cho, I. and D. Kreps (1987). Signaling games and stable equilibria. *The Quarterly Journal of Economics 102*(2), 179–221.

Cintolesi, A. (2020). Political polarization and primary elections. *Working Paper*.

Crutzen, B. S., M. Castanheira, and N. Sahuguet (2009). Party organization and electoral competition. *The Journal of Law, Economics, & Organization 26*(2), 212–242.

Eguia, J. X. and F. Giovannoni (2019). Tactical extremism. *American political science review 113*(1), 282–286.

Erikson, R. S. and G. C. Wright (1980). Policy representation of constituency interests. *Political Behavior 2*(1), 91–106.

Erikson, R. S. and G. C. Wright (1997). Voters, candidates, and issues in congressional elections. In *Congress reconsidered*, Volume 6. Congressional Quarterly Press Washington, DC.

Fiorina, M. P. (1974). *Representatives, roll calls, and constituencies.* Lexington Books.

Fiorina, M. P. and S. J. Abrams (2008). Political polarization in the american public. *Annu. Rev. Polit. Sci. 11*, 563–588.

Fiorina, M. P., S. J. Abrams, and J. Pope (2006). *Culture war?: The myth of a polarized America.* Longman Publishing Group.

Fiorina, M. P. and M. S. Levendusky (2006). Disconnected: The political class versus the people. *Red and blue nation 1*, 49–71.

Gerber, E. R. and R. B. Morton (1998). Primary election systems and representation. *Journal of Law, Economics, and Organization 14*(2), 304–324.

Grofman, B., O. Troumpounis, and D. Xefteris (2019). Electoral competition with primaries and quality asymmetries. *The Journal of Politics 81*(1), 260–273.

Groseclose, T. (2001). A model of Candidate Location When One Candidate Has a Valence Candidate Advantage. *American Journal of Political Science 45*(4), 862–886.

Hirano, S., S. D. Ansolabehere, J. M. Hansen, and J. M. Snyder Jr (2010). Primary Elections and Partisan Polarization in the U.S. Congress. *Quarterly Journal of Political Science 5*(2), 169–191.

Hummel, P. (2010). Flip-flopping from primaries to general elections. *Journal of Public Economics 94*(11-12), 1020–1027.

Hummel, P. (2013). Candidate strategies in primaries and general elections with candidates of heterogeneous quality. *Games and Economic Behavior 78*, 85–102.

Kartik, N. and R. P. McAfee (2007). Signaling character in electoral competition. *American Economic Review 97*(3), 852–870.

McGhee, E., S. Masket, B. Shor, and N. McCarty (2014). A primary cause of partisanship? nomination systems and legislator ideology. *America Journal of Political Science 58*(2), 337–351.

Meirowitz, A. (2005). Informational Party Primaries and Strategic Ambiguity. *Journal of Theoretical Politics 17*(1), 107–136.

Owen, G. and B. Grofman (2006). Two-stage electoral competition in two-party contests: persistent divergence of party positions. *Social Choice and Welfare 26*(3), 547–569.

Poole, K. T. (2005). The decline and rise of party polarization in congress during the 20th century. *Available at SSRN 1154067*.

Serra, G. (2010). Polarization of what? a model of elections with endogenous valence. *The Journal of Politics 72*(2), 426–437.

Serra, G. (2011). Why primaries? the party's tradeoff between policy and valence. *Journal of Theoretical Politics 23*(1), 21–51.

Serra, G. (2013). When will incumbents avoid a primary challenge? aggregation of partial information about candidates' valence. In *Advances in Political Economy*, pp. 217–247. Springer.

Serra, G. (2015). No polarization in spite of primaries: A median voter theorem with competitive nominations. In *The Political Economy of Governance*, pp. 211–229. Springer.

Serra, G. (2018). The electoral strategies of a populist candidate: Does charisma discourage experience and encourage extremism? *Journal of Theoretical Politics 30*(1), 45–73.

Slough, T., E. A. York, and M. M. Ting (2020). A dynamic model of primaries. *The Journal of Politics 82*(4), 1443–1457.

Snyder, J. M. J. and M. M. Ting (2011). Electoral Selection with Parties and Primaries. *American Journal of Political Science 55*(4), 782–796.

Stokes, D. E. (1963). Spatial models of party competition. *American political science review 57*(2), 368–377.

Stone, W. and E. Simas (2010). Candidate Valence and Ideological Positions in U.S. House Elections. *American Journal of Political Science 54*(2), 371–388.

Takayama, S. (2014). A Model of Two-stage Electoral Competition with Strategic Voters. *Working Paper*, 1–36.

Ware, A. (2002). *The American direct primary: party institutionalization and transformation in the north.* Cambridge University Press.

A Proofs

Proof of Lemma 1

Proof. Suppose that $v_r > v_l$ and $v_r - v_l > r - l$. Then $-|x-l|+v_l < -|x-r|+v_r$ for all x. To see this, notice that $-|x-l|-(-|x-r|) \leq r-l$. Therefore, if $v_r - v_l > r-l$ then $-|x-l|+v_l < -|x-r|+v_r$ always holds. Analogously for $v_l > v_r$ and $v_l - v_r > r - l$. Therefore, all voters prefer the candidate at r (l respectively) and the probability for the general election candidate at l of winning the election is 0 (and 1 respectively). Otherwise, to find z, solve the following equation: $-|z-l|+v_l = -|z-r|+v_r$, which yields: $z = \frac{l+r+v_l-v_r}{2}$. We can again use the expression $-|x-l|+v_l < -|x-r|+v_r$ to see that $u_x(r, v_r) > u_x(l, v_l) \Leftrightarrow x > z$ and the other way around (if z takes an infinite value, all voters prefer the l or the r candidate). In other words, voters with a bliss point x greater than z vote for r, whereas voters with a bliss point lower than z vote for l. This is a consequence of the single-peakedness of the utility function $u_x(y, v_y)$. The l candidate therefore wins the general election whenever the median voter bliss-point μ is located to the left of the indifferent voter bliss-point z. Using the fact that the distribution of the median voter bliss-point follows a uniform distribution in $[-b, b]$, the probability for candidate l of winning the election is equal to $Pr(\mu \leq z) = \frac{z+b}{2b}$ if $z \in [-b, b]$; if $z < -b$, the probability of winning takes the corner solution of 0, and viceversa if $z > b$ it takes the value of 1. □

Proof of Lemma 2

Proof. Consider $W_x(l, \theta) = P_\theta(l)[u_x(l, v(\theta)) - u_x(r, v)] + u_x(r, v)$, with $P_\theta(l) > 0$, and differentiate with respect to l. First, notice that:

$$\frac{\partial P_\theta(l)}{\partial l} = \frac{1}{4b}.$$

Concerning $u_x(l, v(\theta)) - u_x(r, v)$, this can be rewritten as $r - l + v(\theta) - v$ for $x \leq l$ and as $r + l - 2x + v(\theta) - v$ for $x \in (l, r)$. Therefore, the derivative is -1 for $x \leq l$ and 1 for $x \in (l, r)$. Putting this together, for $x \leq l$ we have:

$$\frac{\partial W_x(l, \theta)}{\partial l} = \frac{1}{4b}(r - l + v(\theta) - v) - \frac{2b + l + r + v(\theta) - v}{4b} = -\frac{2b + 2l}{4b}$$

which yields $l = -b$. Notice that this does not depend on the valence of candidates, therefore the solution is the same when maximizing $\mathbb{E}_\theta[W_x(l, \theta)]$. Therefore, for all voters such that $x \leq -b$ (i.e. all primary election voters), the optimal platform for a primary election candidate, fixing beliefs $\nu(l)$ to a constant, is $-b$. Notice that the second order condition is negative, so the point found is a maximum. □

Proof of Proposition 1

Proof. I want to show that, given the functional form $u_x(l, v(\theta))$ and Assumption 1, the median voter theorem holds for the primary election (as well as for the general, as it was shown in Lemma 1). That is, the preference for the median voter in the primary election for one or the other candidate determines the outcome of the election.

First of all, consider $\mathbb{E}_{\theta_1}[W_x(l_1, \theta_1)] - \mathbb{E}_{\theta_2}[W_x(l_2, \theta_2)]$ and notice that is is a continuous function.

To simplify the exposition of the proof I consider the case in which the valence of pre-candidates is known (as in a separating equilibrium). Therefore, from this point on I will consider $W_x(l_1, \theta_1) - W_x(l_2, \theta_2)$. The case with uncertainty over pre-candidates' valence is considered at the end of the proof and also goes through.

There are four types of voter x to consider: $x \leq l_1$, $l_1 < x \leq l_2$, $l_2 < x < r$, $x \geq r$. Notice that for the purposes of the primary elections in my model, voters located to the right of r are not relevant.

- Consider voters $x \leq l_1$ first. $W_x(l_1, \theta_1) - W_x(l_2, \theta_2)$ can be rewritten in the following way:

$$\begin{aligned} &\{P_{\theta_1}(l_1)[-l_1 + x + v(\theta_1)] + (1 - P_{\theta_1})[-r + x + v_r]\} \\ &\quad - \{P_{\theta_2}(l_2)[-l_2 + x + v(\theta_2)] + (1 - P_{\theta_2})[-r + x + v_r]\} \end{aligned} \tag{A.1}$$

  which can be rearranged to:

$$P_{\theta_1}(l_1)[r - l_1 + v(\theta_1) - v_r] - P_{\theta_2}(l_2)[r - l_2 + v(\theta_2) - v_r] \tag{A.2}$$

  Notice that this does not depend on x. Therefore, $W_x(l_1, \theta_1) - W_x(l_2, \theta_2)$ is either positive or negative for all these voters, and if any one of these voters is indifferent, they all are.

- Consider now voters $l_1 < x \leq l_2$. $W_x(l_1, \theta_1) - W_x(l_2, \theta_2)$ can be rewritten in the following way:

$$\begin{aligned} &\{P_{\theta_1}(l_1)[l_1 - x + v(\theta_1)] + (1 - P_{\theta_1})[-r + x + v_r]\} \\ &\quad - \{P_{\theta_2}(l_2)[-l_2 + x + v(\theta_2)] + (1 - P_{\theta_2})[-r + x + v_r]\} \end{aligned} \tag{A.3}$$

which can be rearranged to:

$$P_{\theta_1}(l_1)[r + l_1 + v(\theta_1) - v_r - 2x] - P_{\theta_2}(l_2)[r - l_2 + v(\theta_2) - v_r] \tag{A.4}$$

Notice that for these voters, $W_x(l_1, \theta_1) - W_x(l_2, \theta_2)$ is decreasing in x. Therefore, if $W_x(l_1, \theta_1) - W_x(l_2, \theta_2) > 0$ for $x \leq l_1$, there might be a voter indifferent between the two candidates in the interval $(l_1, l_2]$.

- Consider now voters $l_2 < x < r$. $W_x(l_1, \theta_1) - W_x(l_2, \theta_2)$ can be rewritten in the following way:

$$\begin{aligned} &\{P_{\theta_1}(l_1)[l_1 - x + v(\theta_1)] + (1 - P_{\theta_1})[-r + x + v_r]\} \\ &\quad - \{P_{\theta_2}(l_2)[l_2 - x + v(\theta_2)] + (1 - P_{\theta_2})[-r + x + v_r]\} \end{aligned} \tag{A.5}$$

which can be rearranged to:

$$P_{\theta_1}(l_1)[r + l_1 + v(\theta_1) - v_r - 2x] - P_{\theta_2}(l_2)[r + l_2 + v(\theta_2) - v_r - 2x] \tag{A.6}$$

For this group of voters, the derivative of $W_x(l_1, \theta_1) - W_x(l_2, \theta_2)$ with respect to x is $2[P_{\theta_2}(l_2) - P_{\theta_1}(l_1)]$, which can be positive or negative depending on whether $P_{\theta_2}(l_2)$ is larger or smaller than $P_{\theta_1}(l_1)$. If the derivative is positive, then it might be possible to have an indifferent voter in (l_2, r) even when such a voter exists also in $(l_1, l_2]$.

- Finally, consider voters $x \geq r$. Similarly to voters with $x \leq l_1$, $W_x(l_1, \theta_1) - W_x(l_2, \theta_2)$ can be rewritten in the following way:

$$P_{\theta_1}(l_1)[-r + l_1 + v(\theta_1) - v_r] - P_{\theta_2}(l_2)[-r + l_2 + v(\theta_2) - v_r] \tag{A.7}$$

Also for these voters, as for voters with $x \leq l_1$, $W_x(l_1, \theta_1) - W_x(l_2, \theta_2)$ is constant.

From the analysis of $W_x(l_1, \theta_1) - W_x(l_2, \theta_2)$ we can see that there can be at most two indifferent voters, excluding of course the (non-generic cases in which the voters at l_1 or r are indifferent, in which case all voters with $x < l_1$ or $x > r$ respectively are indifferent, too): one is located in $(l_1, l_2]$, the other one in (l_2, r). Moreover, in order for there to be two indifferent voters, a necessary condition is that for voters in (l_2, r), $W_x(l_1, \theta_1) - W_x(l_2, \theta_2)$ is increasing in x. In other words, for there to be two indifferent voters it is necessary that $P_{\theta_2}(l_2) > P_{\theta_1}(l_1)$.

Moreover, suppose we have an indifferent voter in (l_2, r). Denote this voter by $\bar{x}$. The indifferent voter is such that condition (A.6) is equal to zero. This can be rewritten in the following way, noticing that $l_1 + r + v(\theta_1) - v_r = 2z_1$ and $l_2 + r + v(\theta_2) - v_r = 2z_2$, where z_1 and z_2 denote the voters indifferent between l_1 and r and l_2 and r respectively:

$$P_{\theta_1}(l_1)(z_1 - \bar{x}) - P_{\theta_2}(l_2)(z_2 - \bar{x}) = 0 \tag{A.8}$$

In order for (A.8) to hold, it has to be the case that either $\bar{x} < z_1$ and $\bar{x} < z_2$, or that viceversa $\bar{x} > z_1$ and $\bar{x} > z_2$. In other words, the indifferent voter has to either prefer both l_1 and l_2 to r, or prefer r to both l_1 and l_2. However, notice that the former case is impossible: since $P_{\theta_2}(l_2) > P_{\theta_1}(l_1)$, then $z_2 > z_1$. Therefore, suppose that $x < z_1 < z_2$. Given $z_1 - \bar{x} < z_2 - \bar{x}$, it cannot be the case that $P_{\theta_1}(l_1)(z_1 - \bar{x}) = P_{\theta_2}(l_2)(z_2 - \bar{x})$, since the right-hand side has to be larger. Therefore, we are left with the case $z_1 < z_2 < \bar{x}$. Assume now that $\bar{x} \leq -b$. If that were the case, since $\mu \geq -b$, we would have $P_{\theta_2}(l_2) = 0$, and therefore $P_{\theta_1}(l_1) < 0$, which is clearly impossible. Therefore, $\bar{x} > -b$, which means that, given Assumption 1, there can be at most one indifferent voter in the primary election and the median voter theorem holds. In particular, given two candidates l_1 and l_2, l_1 has the support, if of anyone, of voters sufficiently to the left in the primary electorate.

Consider now the general case where the valence of the pre-candidates is not known. Suppose there were two indifferent primary election voters to the left of $-b$. Following the same logic as in the case of certain valence, the largest of their bliss-points has to lie in $(l_2, r]$. Denoting by α and β the probability that pre-candidate 1 and respectively pre-candidate 2 is of type $\theta = A$, the difference in expected utilities from the two candidates for a voter in $(l_2, r]$ can be rewritten as:

$$\begin{aligned} &\alpha \{P_A(l_1)[l_1 - x + v(A)] + (1 - P_A)[-r + x + v_r]\} \\ &\quad + (1 - \alpha) \{P_D(l_1)[l_1 - x + v(D)] + (1 - P_D)[-r + x + v_r]\} \\ &\quad - \beta \{P_A(l_2)[l_2 - x + v(A)] + (1 - P_A)[-r + x + v_r]\} \\ &\quad - (1 - \beta) \{P_D(l_2)[l_2 - x + v(D)] + (1 - P_D)[-r + x + v_r]\} \end{aligned} \tag{A.9}$$

This can be solved for x to yield the location of the voter in $(l_2, r]$ (if it exists) indifferent between the two pre-candidates:

$$\bar{x} = \frac{(l_2 - l_1)(2b + l_2 + l_1 + 2r) + 2v[\beta(b + l_2 + r) - \alpha(b + l_1 + r) - (l_2 - l_1)] + (\alpha - \beta)v^2}{2[l_2 - l_1 - (\alpha - \beta)v]} \tag{A.10}$$

Again following the same logic as in the certain valence case, notice that in order for an indifferent voter to exist in the interval $(l_2, r]$, a necessary but not sufficient condition is that the difference in expected utilities is increasing in x, which requires:

$$\beta P_A(l_2) + (1-\beta)P_D(l_2) > \alpha P_A(l_1) + (1-\alpha)P_D(l_1) \tag{A.11}$$

which can be rewritten as:

$$l_2 - l_1 - (\alpha - \beta)v > 0 \tag{A.12}$$

With condition (A.12) in mind, taking a derivative of (A.10) with respect to r we can immediately see that the bliss-point of the indifferent voter in $(l_2, r]$ in increasing in r. Given that $r \geq 0$, therefore, in order to prove the theorem it is enough to show that $\bar{x} > -b$ evaluating (A.10) at the lower bound for r, that is $r = 0$. In the meantime, equation (A.10) can be simplified with some algebra and rewritten as:

$$\bar{x} = -\frac{v}{2} + b + \frac{(l_2 - l_1)(l_2 + l_1 + 2r) + 2v[\beta(l_2 + r) - \alpha(l_1 + r)]}{2[l_2 - l_1 - (\alpha - \beta)v]} \tag{A.13}$$

Evaluating this at $r = 0$ yields:

$$\bar{x} = -\frac{v}{2} + b + \frac{(l_2 - l_1)(l_2 + l_1) + 2v[\beta l_2 - \alpha l_1]}{2[l_2 - l_1 - (\alpha - \beta)v]} \tag{A.14}$$

Notice that in order for the indifferent voter $\bar{x}$ to have a bliss-point lower than $-b$, it has to be the case that $l_2 < -b < 0$. Focus on $\frac{(l_2-l_1)(l_2+l_1)+2v[\beta l_2 - \alpha l_1]}{2[l_2-l_1-(\alpha-\beta)v]}$. First of all notice that $(l_2 - l_1)(l_2 + l_1) < 0$. Now, if $\beta l_2 - \alpha l_1 \leq 0$, too, the result follows, since $-\frac{v}{2} + b > -b$ is always satisfied given the assumption of $v < b$. Suppose therefore that $\beta l_2 - \alpha l_1 > 0$. If that is the case, then differentiating (A.14) with respect to β and α, we obtain a negative and positive derivative respectively. This means that it is enough to evaluate the condition at $\beta = 1$ and $\alpha = 0$. However, this case falls under the scenario with certain valence that has been demonstrated above, showing that in that case $\bar{x}$ has to be larger than $-b$.

□

Proof of Lemma 3

Proof. To prove this Lemma it is enough to show that $\frac{2r+b+l_1+v-v_r}{2r+b+l_2+v-v_r}$ is increasing in v. Differentiating with respect to v yields: $\frac{l_2-l_1}{(2r+b+l_2+v-v_r)^2} > 0$, delivering the result. □

Lemma A.1. *Consider an equilibrium where a set of policy platforms $\mathcal{L}_{off}$ is off the equilibrium path. If an equilibrium exists under some off-equilibrium beliefs $\nu(l_{off})$ for $l_{off} \in \mathcal{L}_{off}$, then it exists under $\nu(l_{off}) = 0$.*

Proof of Lemma A.1

Proof. Suppose that this was not the case. This would imply that there exists an equilibrium which, changing off-equilibrium beliefs from $\nu(l_{off})$ to 0, does not survive. However, this is impossible, since any deviation becomes less profitable by decreasing beliefs from $\nu(l_{off})$ to 0. The reason is that $\mathbb{E}_{\nu(l_{off})} W_m(l_{off}, \theta_{l_{off}}) \geq W_m(l_{off}, D)$, and strictly if $\nu(l_{off}) > 0$. Therefore, if all deviations to off-equilibrium policies were not profitable under $\nu(l_{off})$, they are also not profitable under $\nu(l_{off}) = 0$. Hence, an equilibrium that exists under $\nu(l_{off})$ will also exist under $\nu(l_{off}) = 0$. □

Proof of Proposition 2

Proof. For the if part, suppose that condition (12) is satisfied at platform l^{pool}. Then, $P^{pr}(-b, l^{pool}) = 0$. Therefore, a pre-candidate deviating to $-b$ would have a payoff of 0, given beliefs $\nu(-b) = 0$. Since by Lemma 2 we have that for all l and $x \leq -b$, $W_x(l, D) \leq W_x(-b, D)$, and given that we are assuming $\nu(l) = 0$ for all other platforms $l \neq l^{pool}$, deviating to any other policy platform delivers a zero payoff, too. Therefore, condition (10) is satisfied and l^{pool} can be part of a pooling equilibrium. For the only if part, I proceed by contradiction. Suppose that in a candidate pooling equilibrium, condition (12) is not satisfied. Then, $P^{pr}(-b, l^{pool}) = 1$. I want to show that in this case, condition (10) is not satisfied, and therefore there exists a profitable deviation. Condition (10) is violated if and only if: $\frac{P_\theta(-b)}{P_\theta(l^{pool})} > \frac{1}{2}$ for at least one type $\theta \in \{A, D\}$. I check it for $\theta = A$, because of the result in Lemma 3. Using Lemma 1 to write the expressions for $P_\theta(l^{pool})$ and solving yields $l^{pool} < r$. Therefore, for any $l^{pool} < r$, if (12) does not hold, then (10) does not hold and pre-candidates have a profitable deviation to $-b$. It remains therefore to be established that $l^{pool} \geq r$ is impossible. To see this, notice that by definition of $w_x(l, \theta)$, $w_x(r, A) = 0$ for all x, and therefore $\mathbb{E}_\theta[W_m(l^{pool} = r, \theta)] < 0$. Following Assumption 1 and Assumption 2, however, $W_m(-b, D) > 0$, from which it immediately follows that (12) is not satisfied. This also allows me to conclude that the upper interval of the interval of pooling equilibria defined by (12) is such that $\bar{l}^{pool} < r$. An analogous reasoning yields that $\underline{l}^{pool} < -b$. Finally, to see that condition (12) indeed defines an interval $[\underline{l}^{pool}, \bar{l}^{pool}]$, notice that by Lemma 2 $\mathbb{E}[W_x(l^{pool}, \theta)]$ is continuous and single-peaked. □

Proof of Lemma 4

Proof. Suppose we have a separating equilibrium in which $l'_D \neq -b$. By Lemma 2, compared to a non-valent pre-candidate at $l'_D \neq -b$, a majority of primary voters prefers a non-valent pre-candidate at $-b$. Deviating away from $l_D^{'}$ therefore results in a probability of winning the primary election of at least $1-\alpha$. Such a deviation is profitable as long as:

$$\frac{1-\alpha}{2}\frac{2b+r+l_d-v}{4b} < (1-\alpha)\frac{2b+r-b-v}{4b}$$

which can be rearranged to:

$$l_D^{'} < r-v$$

Notice that l_D can never be in $[r-v, r+v]$, since a non-valent pre-candidate would never win the general election. Moreover, l_D can also never be to the right of $r+v$, since in that case a deviation to any platform closer to $r+v$ would be profitable. □

Proof of Proposition 3

Proof. By Lemma 4, $l_D = -b$. Following Definition 2 of a Perfect Bayesian Equilibrium, it must be the case that, given the behavior of primary and general election voters, no pre-candidate has a profitable deviation.

I start by showing that condition (16) is necessary for an equilibrium. Suppose we have a separating equilibrium with (l_D, l_A) in which (16) is not satisfied. Then, $\mathbb{E}_{\theta_2}\left[P^{pr}(l_D, l_2(\theta_2))\right] = 1 - \frac{1-\alpha}{2} = \frac{1+\alpha}{2}$. This makes a deviation by a valent pre-candidate to $l_D = -b$ profitable as long as:

$$\frac{\alpha}{2}P_A(l_A) < \frac{1+\alpha}{2}P_A(-b) \tag{A.15}$$

This can be solved to yield: $l_A < \frac{b+r}{\alpha} - b$, where $\frac{b+r}{\alpha} - b \geq r$, strictly for $\alpha < 1$. Therefore, for all $l_A < r$, in a separating equilibrium condition (16) has to be satisfied. Finally, notice that we cannot have $l_A \geq r$ in a separating equilibrium, since valent pre-candidates would have a profitable deviation to $-b$. To see this, notice first that $W_m(l, A) \leq 0$ for all $l \geq r$, whereas $W_m(-b, D) > 0$. Therefore, a valent candidate deviating to $-b$ would win the primary election with probability $1 - \frac{1-\alpha}{2} = \frac{1+\alpha}{2}$. Moreover, the probability of winning the general election with any policy $l \geq r$ is at most $1/2$, whereas the probability of winning the general election at $-b$ is at least $1/4$. Therefore:

$$\frac{\alpha}{2}\frac{1}{2} < \frac{1+\alpha}{2}\frac{1}{4} \Leftrightarrow \alpha < 1,$$

which is always satisfied, meaning that valent candidate indeed have a profitable deviation. It follows that, in a separating equilibrium:

$$\mathbb{E}_{\theta_2}\left[P^{pr}(l_D, l_2(\theta_2))\right] = \frac{1-\alpha}{2}$$

and

$$\mathbb{E}_{\theta_2}\left[P^{pr}(l_D, l_2(\theta_2))\right] = 1 - \frac{1-\alpha}{2} = \frac{2-\alpha}{2},$$

which are used in expressions (17) and (18) to get $a = \frac{1-\alpha}{2-\alpha}$.

I now consider condition (17). I first show that it is a necessary condition for a separating equilibrium to exist. Suppose it is violated: then non-valent pre-candidates have a profitable deviation to platform l_A, contradicting the definition of separating equilibrium. To show instead that (17) is sufficient for non-valent pre-candidates to have no profitable deviation, notice that since $\nu(l) = 0$ for all $l \neq l_A$, deviating to any policy platform other than l_A gives a pre-candidate a zero payoff, since for all $l \notin \{l_A, l_D\}$, $W_m(-b, D) > W_m(l, D)$.

Similarly, if (18) is violated, then valent pre-candidates have a profitable deviation to platform l_D, again contradicting the definition of a separating equilibrium. To show that (18) is sufficient to prevent deviations by valent pre-candidates, notice that deviating to any point other than $-b$ would deliver a payoff of zero, since $W_m(-b, D) > W_m(l, D)$ for all $l \neq -b$.

To sum up, given that by definition of separating equilibrium voters and pre-candidates need to have no profitable deviations, conditions (16), (17) and (18) have to be satisfied. By definition, as explained in the text, this means that $l_A \in \mathcal{IC}^D_{-b} \cap \mathcal{IC}^A_{-b} \cap \mathcal{P}^A_{-b}$. □

Proof of Proposition 4

Proof. Condition (17) is satisfied for platforms such that $l \leq l^e_A$, where l^e_A solves condition (17) as equality, and platforms such that $l \in [r-v, r+v]$, since by Lemma 1, $P_D(l) = 0$ for $l \in [r-v, r+v]$, hence (17) is satisfied.

Consider first platforms $l \leq l^e_A$. First, notice that $l^e_A \leq -b$. By the single-crossing property in Lemma 3, l^e_A also satisfies (18). Solving condition (18) as equality yields a platform $\underline{l}^e_A < l^e_A$, and solving condition (16) as equality gives two solutions, the smaller of which is $\underline{l}^m_A$. The lower bound of the interval of extremist separating equilibria is thus $\max\{\underline{l}^m_A, \underline{l}^e_A\} = \underline{l}_A$. If $\underline{l}_A \leq l^e_A$, there exists an interval of separating equilibria with $l_A \in [\underline{l}_A, l^e_A]$. If instead $\underline{l}_A > l^e_A$, no separating equilibrium exists with $l_A \leq l^e_A$.

Consider now policies $l \in [r-v, r+v]$. First of all, only policies in $[r-v, r)$ need to be considered, since no separating equilibrium can have $l_A \geq r$, as I have shown in Proposition 3. Denote by $l^c_A = r - v$. All these policy-platforms also satisfy (18), since

$P_A(l) > P_A(-b)$ for all $l \in [r-v, r)$. Finally, denote by $\bar{l}_A^m$ the largest of the two solutions to condition (16). If $\bar{l}_A^m < l_A^c$, then no platform in $[r-v, r)$ is in $\mathcal{P}_{-b}^A$, hence no separating equilibrium with $l_A \in [r-v, r)$ exists. If instead $\bar{l}_A^c \geq l_A^c$, policy platforms $[l_A^c, \bar{l}_A^m]$ can be part of a separating equilibrium. □

Proof of Corollary 1

Proof. Suppose condition (16) is satisfied at l_A^e. By definition, l_A^e satisfies condition (17). Following Lemma 3, condition (18) is satisfied, too. From Proposition 3, a separating equilibrium with $l_A = l_A^e$ exists. For the only if part, suppose that condition (16) is not satisfied at l_A^e. Since all other values of $l_A < -b$ satisfying condition (17) are smaller than l_A^e, so for these values of l_A, $W_m(l_A, A) < W_m(l_A^e, A)$ and therefore they also fail to satisfy condition (16). Following Proposition 3, no extremist separating equilibrium exists.

I now consider centrist separating equilibria. Suppose condition (16) is satisfied at l_A^c. Since $l_A^c \geq -b$, (18) is satisfied. Given that condition (17) is also satisfied at l_A^c, we can conclude that a centrist separating equilibrium exists. For the only if part, suppose that (16) is not satisfied at l_A^c. Then all other candidate l_A in $(r-v, r+v]$ fail to satisfy (16), given that $W_m(l_A, A) < W_m(l_A^c, A)$. Following Proposition 3, no cenrtrist equilibrium exists. □

Proof of Proposition 5

Proof. This proposition is a direct corollary of Lemma 2. Take the pooling equilibrium. Since for all l^{pool}, $\nu(l^{pool}) = \alpha$, by Lemma 2 the optimal pooling candidate for all party voters is located at $-b$. Similarly, for the separating equilibria, the expected utility from a separating equilibrium can be written as:

$$\alpha(2-\alpha)W_x(l_A, A) + (1-\alpha)^2 W_x(l_D, D) \tag{A.16}$$

Since $l_D = -b$ in all separating equilibria and the probability of selecting a valent pre-candidate is constant at $\alpha(2-\alpha)$ across all separating equilibria, comparing two separating equilibria with l_A and $l_A^{'}$ reduces to comparing $W_x(l_A, A)$ and $W_x(l_A^{'}, A)$. Since $\nu(l_A) = \nu(l'_A) = 1$ for all feasible values of l_A and l'_A, Lemma 2 applies. Hence, among all extremist separating equilibria, with $l_A \in [\underline{l}_A, l_A^e]$, the welfare maximizing is that at l_A^e since it is closest to $-b$. Similarly, among all centrist separating equilibria, with $l_A \in [l_A^c, \bar{l}_A^c]$, the welfare maximizing is the one with $l_A = l_A^c$. □

Proof of Proposition 6

Proof. For the centrist separating equilibrium, the results follows immediately from $l_A^c = r - v$. For the extremist separating equilibrium, it follows from expression (19), i.e. $l_A^e = -b - (1-a)(b + r - v)$. Notice that since $a = \frac{1-\alpha}{2-\alpha}$, $\frac{\partial a}{\partial \alpha} < 0$. □

Proof of Proposition 7

Proof. A majority of primary election voters prefers the extremist to the centrist separating equilibrium if and only if

$$\alpha(2-\alpha)W_m(l_A^c, A) + (1-\alpha)^2 W_m(l_D, D) \leq \alpha(2-\alpha)W_m(l_A^e) + (1-\alpha)^2 W_m(l_D, D) \quad \text{(A.17)}$$

which yields:

$$W_m(l_A^c, A) \leq W_m(l_A^e, A). \quad \text{(A.18)}$$

The latter condition can be rewritten as:

$$P_A(l_A^c)[r - m - (l_A^c - m)] \leq P_A(l_A^e)[r - m - |l_A^e - m|] \quad \text{(A.19)}$$

which can be rearranged to yield condition (24). Notice that the left-hand side does not depend on m, whereas the right-hand side is weakly decreasing in m. Therefore, if (A.19) holds for some values of $m \geq l_A^e$, then it holds for all $m < l_A^e$, and if it does not hold for any $m \geq l_A^e$, then it does not hold for any $m < r$. Hence, consider $m \geq l_A^e$ and rewrite (A.19) as:

$$m \leq \frac{r + l_A^e}{2} - \frac{v}{2}\frac{P_A(l_A^c)}{P_A(l_A^e)} \quad \text{(A.20)}$$

which is indeed condition (23). □

Proof of Proposition 8

Proof. The following conditions represent the welfare comparisons between equilibria: first of all, condition (24) (or (A.19) in the proof above) determine the set of parameters such that the extremist equilibrium is preferred to the centrist separating. Concerning the comparison between the centrist separating and the pooling, the condition is:

$$\begin{aligned} &\alpha(2-\alpha)P_A(l_A^c)[r - m - (l_A^c - m)] + (1-\alpha)^2 P_D(-b)(r + b - v) \\ &\quad \geq \alpha P_A(-b)(r + b) + (1-\alpha)P_D(-b)(r + b - v) \end{aligned} \quad \text{(A.21)}$$

whereas the condition comparing the extremist separating equilibrium and the pooling is:

$$\begin{aligned} &\alpha(2-\alpha)P_A(l_A^e)[r - m - |l_A^e - m|] \\ &\quad + (1-\alpha)^2 P_D(-b)(r + b - v) \geq \alpha P_A(-b)(r + b) + (1-\alpha)P_D(-b)(r + b - v) \end{aligned} \quad \text{(A.22)}$$

where $P_A(l_A^c) = \min\left\{\frac{2b+2r-v}{4b}, 1\right\}$ and $P_A(l_A^e) = \frac{a(b+r)+(1-a)v}{4b}$. Evaluating inequalities (A.21) and (A.22) using a solver yields bounds on v, r or b above which (in the case of v) or below which (in the case of r and b) every separating equilibrium is preferred to the pooling. □

Definition A.1. *In order to apply to my game the D1 criterion introduced by Cho and Kreps (1987), I compare how different types profit after a deviation for all possible mixed responses by primary election voters. In my setup, this means, for a given deviation from the equilibrium path, to look for the type that would profit from a deviation under a lower probability of winning the primary election. The minimum probability of winning the primary election that makes a deviation profitable is defined as follows. For any equilibrium policy platform l^*, a deviation to l' is profitable for a pre-candidate of type θ if the probability of winning the primary election is at least $\pi_\theta(l', l^*)$ defined by:*

$$\pi_\theta(l', l^*)P_\theta(l') > \mathbb{E}_{\theta_2}\left[P^{pr}(l^*, l_2(\theta_2))\right] P_\theta(l^*) \tag{A.23}$$

Following a deviation from l^* to some l', voters assign probability 1 *to the type θ associated to the minimum $\pi_\theta(l', l^*)$.*

Proof of Proposition 9

Proof. First, I show that the D1 criterion, as per Definition A.1, destroys all pooling equilibria. To do this, notice that, when deviating out of a pooling equilibrium, $\pi_\theta(l', l^*)$ is denoted by:

$$\pi_\theta(l', l^{pool}) = \frac{1}{2}\frac{P_\theta(l^{pool})}{P_\theta(l')}$$

From Lemma 3 it follows immediately that, for any $l' < l^{pool}$, $\pi_A(l', l^{pool}) < \pi_D(l', l^{pool})$. Therefore, any deviation to $l' < l^{pool}$ would be interpreted as coming from a valent type. Given that $P_A(l^{pool}) > 0$ and it is continuous in l, deviating to l' sufficiently close to l^{pool} is profitable and would allow the deviating pre-candidate to win the primary election with probability 1. As a result, no pooling equilibrium is consistent with the D1 criterion.

Second, I show that all separating equilibria of the centrist type except for that at l_A^c are eliminated by the D1 criterion. Since $P_D(l) = 0$ for all $l \in [r-v, r+v]$, a deviation to such platforms can never be profitable for a low type. As a result, $\nu(l) = 1$ for all $l \in [r-v, r+v]$. Suppose that a centrist separating equilibrium was played, in which $l_A = \tilde{l} in (r-v, \bar{l}_A^m]$. Any valent pre-candidate deviating to some $l' \in [r-v, \bar{l}_A^m]$, subject to $l' < \tilde{l}$ would win the primary elections with probability one, since $W_m(l', A) > W_m(\tilde{l}, A) \geq W_m(-b, D)$. Moreover, for l' close enough to $\tilde{l}$, such a deviation is profitable to a valent pre-candidate, by continuity of $P_A(l)$. Therefore, all centrist separating equilibria with $l_A > l_A^c$ are destroyed under the D1 criterion.

Finally, I analyze the case of extremist separating equilibria. I want to show that under the D1 criterion, all such separating equilibria except for the one with $l_A = l_A^e$, are destroyed. Suppose that an extremist separating equilibrium with $l_A = \tilde{l} < l_A^e$ is played. Following the D1 criterion, there is a set of policy platforms l' such that $\nu(l') = 1$. In particular, following Definition A.1, this set consists of policies l' such that:

$$\frac{P_D(l')}{P_D(l_D)} < a\frac{P_A(l')}{P_A(\tilde{l})} \tag{A.24}$$

Notice that, for any $\tilde{l} < l_A^e$, a deviation to $l' = l_A^e$ satisfies condition (A.24), which in that case reduces to $P_A(\tilde{l}) < P_A(l_A^e)$. Since a deviation to l_A^e is profitable for valent types, this means that any separating equilibrium with $l_A = \tilde{l} < l_A^e$ cannot survive the refinement applied by D1. □

Proof of Proposition 10

Proof. If the welfare dominant equilibrium is pooling, it cannot be the outcome of the game refined with the D1 criterion. Following Proposition 9, as a matter of fact, all pooling equilibria are destroyed by D1.

Let's now consider the centrist equilibrium. Notice that following the D1 criterion, $\nu(l) = 1$ for all platforms $l \in [r - v, r + v]$, independently of the equilibrium played. Therefore, $\nu(r - v) = 1$. Suppose the centrist separating equilibrium is welfare dominant. Consider at first deviations to some $l' > r - v$: following such a deviation, a pre-candidate can at most win the primary election when no other valent candidate is drawn. In order to rule out such deviations, I need to evaluate the best possible among them, i.e. the one to $\bar{l}_A^m$, that is the right-most policy platform such that a valent pre-candidate can still win the primary against a non-valent opponent. It follows that deviating to any $l > \bar{l}_A^m$ yields a payoff of zero, since a majority of primary voters prefer the non-valent candidate at $-b$. This yields a condition for the centrist separating equilibrium at l_A^c to survive the D1 criterion. This condition is simply condition (26) evaluated at $l_A = l_A^c$:

$$(1 - \alpha)P_A(\bar{l}_A^m) \leq \left(1 - \frac{\alpha}{2}\right) P_A(l_A^c) \tag{A.25}$$

Notice that for sufficiently low α, deviating to $\bar{l}_A^m$ is always profitable.

Concerning deviations to policies to the left of l_A^c, a sufficient condition that can be checked is that given l_A^c, no policy that a majority of primary voters prefer to l_A^c has beliefs restricted to $\nu = 1$. Following the D1 criterion, there is an upper bound l'_A such that for $l \leq l'_A$, $\nu(l) = 1$. However, $l'_A < l^e_A$. To see this, I can write the expression for the thresholds $\pi_\theta(l^e_A, l^*_\theta)$ which gives:

$$a = \frac{P_D(l^e_A)}{P_D(l_D)} > a\frac{P_A(l^e_A)}{P_A(l^c_A)} \tag{A.26}$$

As a result, there is no profitable deviation to the left of l^c_A, which allows me to concluse that, when it is welfare dominant, the centrist separating equilibrium exists as long as condition (A.25) is satisfied.

I now want to show that the extremist separating equilibrium does not exist if the centrist separating is welfare dominant. If the extremist separating equilibrium is played, a candidate deviating to $r - v$ would win the primary with probability one, since the centrist separating equilibrium is welfare dominant. This deviation is profitable for valent pre-candidates, since $l^c_A > l^e_A$. Hence, an extremist separating equilibrium cannot exist. Therefore, if condition (A.25) is satisfied, then the centrist separating equilibrium is the only one selected by the D1 criterion.

I now look at the case in which the extremist separating equilibrium at l^e_A is the welfare dominant equilibrium. Notice that under the D1 criterion, $\nu(l) = 1$ for $l \in [r - v, r + v]$. Condition (26) evaluated at $l_A = l^e_A$ is written as:

$$\frac{2-\alpha}{2} P_A(l^e_A) \geq (1-\alpha) P_A(\bar{l}^c_A) \tag{A.27}$$

and states that a deviation to policy $\bar{l}^c_A$ defined in Proposition 4 is not profitable for a valent pre-candidate starting from the separating equilibrium at l^e_A. If however, condition (A.27) is not satisfied, then a profitable deviation to $\bar{l}^c_A$ exists under the D1 criterion the extremist separating equilibrium at l^e_A does not exist despite being welfare dominant.

Suppose that (A.27) is satisfied: is the extremist separating equilibrium at l^e_A the unique equilibrium selected by the D1 criterion? This requires either the centrist equilibrium not to exist, or to be destroyed by the D1 criterion. The former case requires $W_m(l^c_A) < W_m(l_D - D)$. Suppose instead that the centrist separating equilibrium exists. If the D1 criterion destroys it because condition (26) is not satisfied at l^c_A, then also the extremist separating equilibrium is destroyed, since condition (26) not holding at l^c_A implies that it does not hold at l^e_A, too. Therefore, I need to look at a deviation to the left of l^c_A. The most profitable of such deviations is to l'_A, which is defined by the following condition:

$$\frac{P_D(l'_A)}{P_A(l'_A)} = a\frac{P_D(l_D)}{P_A(l^c_A)} \tag{A.28}$$

Condition (A.28) states that at point l'_A low types benefit from a deviation out of $-b$ for the same set of mixed best replies as high types deviating out of l^c_A. Therefore, for all points $l < l'_A$, $\nu(l) = 1$ when a centrist separating equilibrium is being played. A deviation out of l^c_A to a point arbitrarily close (but smaller) to l'_A is profitable if and only if:

$$P_A(l'_A) > \frac{2-\alpha}{2} P_A(l^c_A). \tag{A.29}$$

□

Proof of Corollary 2

Proof. This result follows directly from Proposition 10 and the equivalence between the pooling equilibrium and direct nomination by party elites. When the pooling equilibrium is welfare dominant, direct party nomination is preferred, since the pooling equilibrium does not survive the D1 criterion. Concerning the second point, this also follows directly from Proposition 10: the D1 criterion always selects the centrist equilibrium when it is welfare dominant, and hence preferred to the pooling equilibrium/direct party nomination. Finally, concerning the third point, again following Proposition 10, it can be the case that the centrist separating equilibrium is selected by the D1 criterion, despite the extremist separating equilibrium being welfare dominant. When this happens, if the pooling equilibrium gives a majority of voters a higher welfare than the centrist equilibrium, then direct nomination is preferred to primaries. □

B Double Primary

In this section I consider an environment in which both parties simultaneously hold primary elections to select their candidates. For simplicity, I model the R party as perfectly symmetric with respect to the L party: the probability of drawing a valent candidate is α for both parties; the primary median voters are located at $m_L \leq -b$ for the L party and $m_R = -m_L \geq b$ for the R party. I still assume that $v \leq b$.

From the point of view of each party, the only difference with respect to the single primary environment is that the policy platform and valence of the opposing party's candidate are uncertain. Therefore, the results of Proposition 1, Lemma 2 and Lemma 3 still hold in this new setup.

In the analysis of the double primary game I focus on symmetric equilibria, that is equilibria in which the the strategies chosen by pre-candidates and voters in equilibrium are invariant to flipping the labels of the two parties. When considering symmetric equilibria in the double primary game, it is useful to have a way to rank equilibria in terms of expected utility they provide to primary election voters in each party. The result is that all voters weakly prefer symmetric equilibria in which candidates are less polarized. Moreover, a crucial variable which positively affects the expected utility from an equilibrium is the probability of electing the valent candidate when a valent candidate faces a non-valent candidate in the general election. As we will see, this implies that centrist separating equilibria are preferred to extremist separating equilibria by all voters, because in a centrist equilibrium a valent candidate, being closer to the center, has a policy advantage on top of the valence advantage when running against a non-valent one in the general election.

Lemma B.1. *Consider a symmetric equilibrium with $l_A = -r_A$ and $l_D = -r_D$ and denote by $\sigma_{\theta,\theta'}$ the probability that candidates of two types θ and θ' face each other in the general election. Denote by ϕ the probability that a valent candidate wins the general election when facing a non-valent candidate from the opposing party. The expected utility from such a symmetric equilibrium, for a voter located at x, is:*

$$\sigma_{A,A}[\min\{l_A, x\} + v] + \sigma_{D,D}\min\{l_D, x\} + 2\sigma_{A,D}[\phi\min\{l_A, x\} + (1-\phi)\min\{l_D, x\} + \phi v] \tag{B.1}$$

Lemma B.1 has some important consequences: first, when comparing symmetric pooling equilibria, with $l_A = l_D = l^{pool}$, for a given ϕ, the closer l^{pool} is to zero the weakly higher is the expected utility for all voters. The same holds for separating equilibria:

fixing $l_D = -b$, the closer to zero l_A the weakly higher expected utility: therefore, centrist equilibria are preferred to extremist equilibria. Moreover, notice that ϕ is also an equilibrium object, since it denotes the probability of electing a valent candidate in a general election between a valent and a non-valent candidate. This provides another reason to prefer centrist to extremist equilibria. Since they are more moderate, in a centrist equilibrium valent candidates have a policy advantage on top of the valence advantage. In an extremist equilibrium, on the other hand, valent candidates have a policy disadvantage that partially offsets their valence advantage. A similar reasoning can be applied to pooling equilibria: in terms of policy, pooling equilibria with l^{pool} closer to zero deliver a weakly higher expected utility (strictly higher for voters with $x < l^{pool}$ and $x > r^{pool}$). In terms of ϕ, for all pooling equilibria with $l^{pool} < -v/2$ the value of ϕ is constant and equal to $\frac{2b+v}{4b}$, whereas for $l^{pool} \geq -v/2$ it takes the value of 1, since $r^{pool} - l^{pool} \leq v$, making these pooling equilibria preferred from the welfare point of view.

I now discuss the symmetric pooling and separating equilibria of the game. Just like in the single-primary game, there is an interval of pooling equilibria. Whereas the welfare dominant ones for primary election voters are those with $\phi = 1$, i.e. with $l^{pool} \geq -v/2$, these pooling equilibria need not exist (whereas for example the pooling equilibrium at $l^{pool} = -b$ exists for all parameters). Moreover, the pooling equilibrium that maximizes the welfare of primary election voters fixing what the opposing party does is the one at $-b$, following Lemma 2.

A similar argument holds for the centrist separating equilibrium. Whereas the best one in terms of welfare is that where valent pre-candidates locate at $l_A = r_A = 0$, the one existing for a the largest set of parameters is that in which $l^c_{dp} = -\frac{v}{2}$ and $r^c_{dp} = \frac{v}{2}$. This is the centrist separating equilibrium I focus on in the analysis, since in any case any centrist separating equilibrium dominates any extremist separating equilibrium in the double primary game.

Finally, concerning extremist separating equilibria, there is a continuum of extremist separating equilibria, among which the one delivering the highest expected utility to voters (primary and general election voters alike) is the one with the least polarized policies, following Lemma B.1. Just like in the single primary game, $l_D = -b$ (and by symmetry $r_D = b$) whereas $l_A = l^e_{dp}$ defined as:

$$l^e_{dp} = -b - \frac{(1-a)}{1-(1-a)\omega}[2b - \omega v] = -b - \underbrace{\frac{1}{(1-\alpha)(2-\alpha)}[2b - \alpha(2-\alpha)v]}_{\equiv \Delta} \tag{B.2}$$

and $r_A = r^e_{dp} = -l^e_{dp}$ analogously defined, where $\omega \equiv \alpha(2-\alpha)$ denotes the probability that the primary of each party selects a valent candidate. Notice that since $v \leq b$ and $\omega \leq 1$, $2b - \omega v > 0$, which guarantees that $r^e_{dp} > b$ and $l^e_{dp} < -b$.

Proposition B.1. *There is an interval of possible symmetric pooling equilibria with* $l^{pool} \in [\underline{l}^{pool}, \bar{l}^{pool}]$. *The double primary game also has an interval of possible symmetric centrist separating equilibria in which valent pre-candidates choose platforms* $l^c_{dp} \in [-v/2, 0]$ *and* $r^c_{dp} = -l^c_{dp}$ *and non-valent pre-candidates choose platforms* $l_D = -b$ *and* $l_D = b$ *respectively. The double primary game also has an interval of possible symmetric extremist separating equilibria with* $l_D = -b = -r_D = b$ *and* $l_A \in [\underline{l}^e_{dp}, l^e_{dp}]$ *and* $r_A = -l_A$.

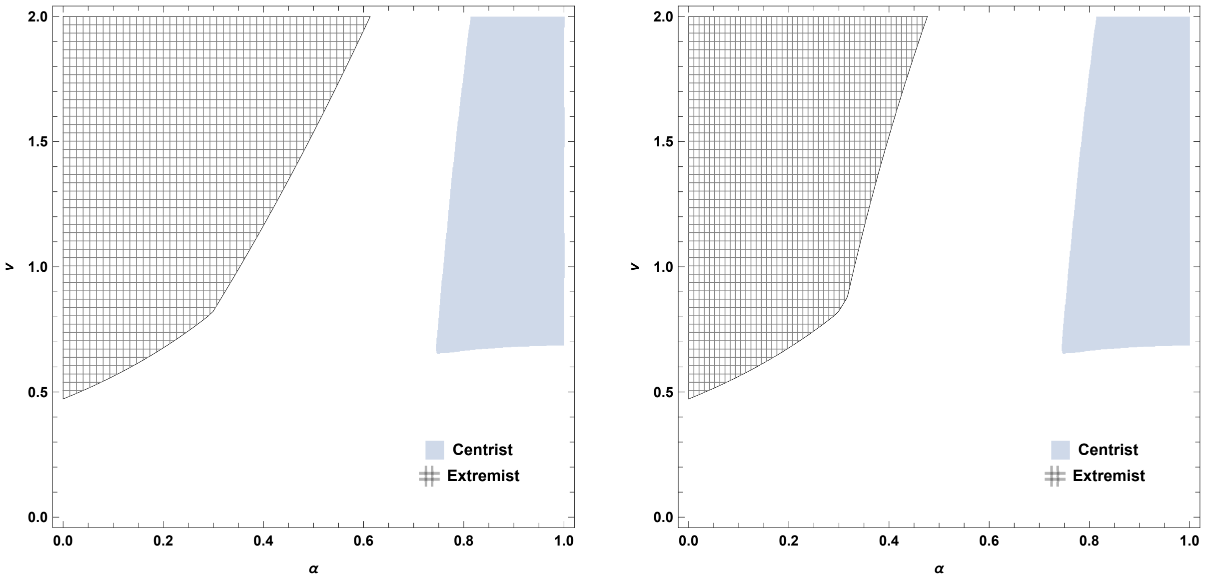
As I already mentioned when discussing Lemma B.1, in the symmetric double primary game all voters prefer a centrist separating equilibrium, when it exists, to the welfare maximizing extremist separating equilibrium. Concerning the ranking between pooling and extremist separating equilibria, in the analysis I compare the welfare maximizing extremist equilibrium, with $l_A = l^e_{dp}$, to the pooling at $-b$ as reference point, not least because it exists for all parameter values.

In Figure B.1a I display the set of parameters for which a centrist separating equilibrium exists (the set coincides with the parameters for which the centrist separating equilibrium with $l_A = -v/2$ exists) and thus is welfare dominant, and the parameter values for which the welfare maximizing extremist separating equilibrium dominates the pooling at $-b$. In the areas without filling, the pooling at $l^{pool} = -b$ dominates the welfare maximizing separating equilibrium.

Unlike in the single primary case, in the double primary game the symmetric centrist equilibrium exists only under a restricted set of parameters: this has to do with the incentive compatibility constraint for non-valent pre-candidates. Notice that the extremist separating equilibrium is preferred when v is high (relative to b) and α is small, whereas some centrist equilibrium exists and is welfare dominant for high v (relative to b) and large α. Notice that for low values of v, separating equilibria do not exist, similar to what happens in the single primary baseline. However, in the double primary game there are intermediate values of α for which the pooling equilibrium is preferred even under high values of v.

Proposition B.2. *The welfare dominant equilibrium of the double primary game is the centrist separating equilibrium, when it exists. If the centrist separating equilibrium does not exist, the extremist separating equilibrium with* $l_A = l^e_{dp}$ *dominates the pooling equilibrium at* $-b$ *when it exists and condition* (B.12) *(presented in the proof of the proposition) is satisfied.*

Figure B.1: Double Primary Game: Symmetric Separating Equilibria



(a) Existence of Separating Equilibria

Figure B.1a shows the parameter space such that the extremist (with $l_A = l^e_{dp} = -r_A$) and centrist (with $l_A = -\frac{v}{2} = -r_A$) separating equilibria exist.

(b) Welfare Dominant Equilibrium

Figure B.1b shows the welfare dominant equilibrium of the game in the (α, v) space for parameter values $m = -5$ and $b = 2$. The pooling equilibrium is welfare dominant when no filling is present.

Finally, a few words on equilibrium selection through the D1 criterion, which I omit for reasons of space. Just like in the single primary case, the D1 criterion destroys all pooling equilibria. Moreover, it allows for centrist or extremist deviations which can destroy the extremist or the centrist separating equilibrium. However, compared to the single primary baseline, it is less likely for a centrist deviation to be profitable, whereas an extremist deviation from a centrist separating equilibrium can be profitable, which was not the case in the single primary baseline.

B.1 Proofs of Double Primary Game

Proof of Lemma B.1

Proof. I consider here voters of the L party primary, for which $x \leq \min\{r_A, r_D\}$ always holds, but the argument is analogous for voters of the R party. A symmetric equilibrium can give rise to three possible scenarios: with probability $\sigma_{A,A}$, depending on the type of equilibrium, two valent candidates are selected, and by symmetry each wins with probability $1/2$. The expected utility conditional on two valent candidates being selected is thus:

$$\frac{1}{2}[-(r_A - x)] + \frac{1}{2}[-|l_A - x|] + v = \begin{cases} x + v \text{ if } x \leq l_A \\ -\frac{1}{2}r_A + \frac{1}{2}l_A + v = l_A + v \text{ if } r_A > x > l_A \end{cases} \tag{B.3}$$

This can be rewritten as:

$$\frac{1}{2}[-(r_A - x)] + \frac{1}{2}[-|l_A - x|] + v = \min\{l_A, x\} + v \tag{B.4}$$

The case of two non-valent candidates, which occurs with probability $\sigma_{D,D}$, is analogous, with l_D substituting l_A, r_D substituting r_A, and $v = 0$. Finally, consider the case of one valent and one non-valent candidate selected by the respective primary elections. This happens with probability $\sigma_{A,D} = \sigma_{D,A}$ in each primary election. Putting together the scenarios in which the L party selected a valent candidate and that in which the R party did, which happens with probability $2\sigma_{A,D}$, yields the following expected utility:

$$\begin{aligned} &\frac{1}{2}\{\phi[-|l_A - x| + v] + (1-\phi)[-(r_D - x)]\} \\ &+ \frac{1}{2}\{\phi[-(r_A - x) + v] + (1-\phi)[-|l_D - x|]\} = -\phi \min\{x, l_A\} - (1-\phi)\min\{x, l_D\} + \phi v \end{aligned} \tag{B.5}$$

Summing up the expected utility from the three scenarios finally yields condition (B.1).

Notice that this lemma is useful to compare the welfare from different types of equilibria. First of all, given everything else, the smaller l_A and l_D, the weakly higher welfare is for all voters. This can clearly be seen from the fact that the expected utility depends on $\min\{x, l_\theta\}$: therefore, polarization hurts voters with $x > l_\theta$. Second, given everything else, equilibria with higher ϕ are preferred. □

Proof of Proposition B.1

Proof. The construction of the separating equilibria of the game is very similar to the single-primary case. The conditions for the L and the R party primaries are completely analogous, given the symmetry of the environment. Therefore, I focus on the conditions for the L party primary here, which allow for a more direct comparability to the single primary case. Consider pooling equilibria first: similar to what happens in the single-primary game, the bounds of the interval of pooling equilibria are defined by:

$$\mathbb{E}_\theta W_m(\underline{l}^{pool}, \theta) = W_m(-b, D) = \mathbb{E}_\theta W_m(\bar{l}^{pool}, \theta) \tag{B.6}$$

I now analyze separating equilibria. First of all, the result equivalent to that of Lemma 4 goes through: in a pure strategy separating equilibrium, non-valent pre-candidates choose platform $l_D = -b$ in the L party and $r_D = b$ in the R party. Second, in a pure strategy separating equilibrium, primary election voters choose a valent pre-candidate whenever possible. This means that in any separating equilibrium:

$$W_m(l_A, A) \geq W_m(l_D, D) \tag{B.7}$$

Consider now centrist separating equilibria: in such equilibria, $l_A \in [r_A - v, r_A]$. Using symmetry we get that $l_A \in [-v/2, 0]$ and $-l_A = r_A \in [0, v/2]$. Unlike in the single-primary baseline, where incentive compatibility with respect to types $\theta = D$ holds for all policies $l_A \in [r - v, r]$, in the double primary game this is not the case. The reason is that if the R party candidate is at $r_D = b$, a non-valent pre-candidate choosing platform $r_A - v$ wins with positive probability (higher than when facing the same candidate having chosen platform l_D). Therefore, condition (13) becomes:

$$\begin{aligned} &\frac{1-\alpha}{2}[\alpha(2-\alpha)P_{D,A}(-b, l_A) + (1-\alpha)^2 P_{D,D}(-b, b)] \\ &\quad \geq \frac{2-\alpha}{2}[\alpha(2-\alpha)\underbrace{P_{D,A}(l_A, r_A)}_{=0} + (1-\alpha)^2 P_{D,D}(l_A, b)] \end{aligned} \tag{B.8}$$

Notice that incentive compatibility for valent pre-candidates is always satisfied. Finally, concerning the extremist separating equilibrium, the upper bound of the interval is given by the incentive compatibility constraint for non-valent pre-candidates, which reads:

$$\begin{aligned} &\frac{1-\alpha}{2}[\alpha(2-\alpha)P_{D,A}(-b, l_A) + (1-\alpha)^2 P_{D,D}(-b, b)] \\ &\quad \geq \frac{2-\alpha}{2}[\alpha(2-\alpha)P_{D,A}(l_A, r_A) + (1-\alpha)^2 P_{D,D}(l_A, b)] \end{aligned} \tag{B.9}$$

Solving (B.9) as equality for both the L and the R party yields:

$$l_A \leq -b - (1-a)[b + \underbrace{(1-\omega)r_D + \omega(r_A - v)}_{\text{Exp. location of } r}]$$

and analogously that for the right candidate:

$$r_A \geq b + (1-a)[b - \underbrace{(1-\omega)l_D - \omega(l_A + v)}_{\text{Exp. location of } l}]$$

Notice that these constraints are the same as those for the single primary election, but instead of r (and respectively l) one has to write the expected location of the opponent, adjusted for valence.

Analogously to the single-primary case, the lower bound of the interval of extremist separating equilibria is either the point l_A where $W_{m_L}(l_A, A) = W_{m_L}(l_D, D)$ or, if larger, in the point where the incentive compatibility constraint for valent pre-candidates, that is the following condition, holds with equality:

$$\begin{aligned} &\frac{2-\alpha}{2}[\alpha(2-\alpha)P_{A,A}(l_A, r_A) + (1-\alpha)^2 P_{A,D}(l_A, b)] \\ &\geq \frac{1-\alpha}{2}[\alpha(2-\alpha)P_{A,A}(-b, r_A) + (1-\alpha)^2 P_{A,D}(-b, b)] \end{aligned} \tag{B.10}$$

Notice that, by Lemma 3, which goes through also in the double primary equilibrium, at the point where (B.9) holds with equality condition (B.10) is strictly satisfied. □

Proof of Proposition B.2

Proof. The fact that any centrist separating equilibrium is more efficient than the welfare maximizing extremist equilibrium is a direct consequence of Lemma B.1. To see it, take x sufficiently low, in such a way that, policy-wise, a voter is indifferent between the two equilibria (more moderate voters have a further reason to prefer the centrist equilibrium), since $\min\{l_\theta, x\} = x$ for all l_θ. First of all, since we are comparing two separating equilibria, $\sigma_{\theta,\theta'}$ are the same in both equilibria for each combination (θ, θ'). The only other object that changes is therefore ϕ, which I denote as ϕ_c in a centrist and ϕ_e in an extremist separating equilibrium. In a centrist separating equilibrium, $\phi_c \geq \frac{2b+b-v/2+v}{4b}$. In the welfare maximizing extremist equilibrium, instead, $\phi_e = \frac{2b+b-b-\Delta+v}{4b}$. Since $\Delta > 0$ and $v \leq b$, $\phi_c > \phi_e$. Therefore, all voters (also non-primary voters) prefer any centrist separating equilibrium to the welfare maximizing extremist separating. When comparing instead a centrist separating equilibrium with a pooling equilibrium, the best pooling equilibrium is policy-wise equivalent to any centrist separating equilibrium for primary voters of both parties, since all primary voters have $|x| > b$. Given ϕ, a separating equilibrium is more efficient at selecting valent pre-candidates in the primary, which gives an advantage to the separating equilibrium. However, in some pooling equilibria $\phi_{pool} = 1$, which happens when $l^{pool} \geq -v/2$. In the centrist separating equilibrium associated with lowest welfare, on the other hand, $\phi_c = \frac{2b+b-v/2+v}{4b}$. Hence, there is a trade-off, which however always goes in favor of the centrist separating equilibrium. As a matter of fact, the following condition always holds, even for $\phi_{pool} = 1$ and $\phi_c = \frac{2b+b-v/2+v}{4b}$, that is its lower bound in a centrist separating equilibrium:

$$[\alpha(2\alpha)]^2 v + 2\alpha(2-\alpha)(1-\alpha)^2\phi_c v \geq \alpha^2 v + 2\alpha(1-\alpha)\phi_{pool} v \tag{B.11}$$

Therefore, the centrist separating equilibrium is welfare dominant, if it exists. Concerning the comparison between the welfare maximizing extremist separating equilibrium and the pooling at $-b$, the condition to consider is the folllwing:

$$\begin{aligned}&[\alpha(2-\alpha)]^2[\min\{l_A, m_L\} + v] + (1-\alpha)^4(-b)\\&\quad + 2\alpha(2-\alpha)(1-\alpha)^2[\phi_e \min\{l_A, m_L\} + (1-\phi_e)(-b) + \phi_e v] \geq m_L\\&\qquad + \alpha^2 v + 2\alpha(1-\alpha)\frac{2b+v}{4b}v\end{aligned} \tag{B.12}$$

□

C Policy Conflict

An alternative scenario to that described in Section 6 is one where there is a policy conflict between the elites and party voters, but elites are capable of identifying valent pre-candidates.

In particular, I now assume that, absent primary elections, party elites would draw two pre-candidates from the pool and make one of them run with policy platform $p \neq -b$, after having observed their valence and selected a valent one, if available. In this scenario, the probability of having a valent candidate is the same under a separating equilibrium following primary elections and under direct nomination by the elites: this allows me to isolate the effect of the policy conflict. Voters prefer primaries if the policy distortion associated with a separating equilibrium is smaller ≥ than the one due to the policy bias of elites[30]. From the point of view of party elites, instead, direct nomination achieves the first best: therefore, elites would never be in favor of primaries, unless they can directly benefit from primaries, for example in terms of improved reputation thanks to a more transparent selection process. With these additions, my model can thus also shed some light on the circumstances under which party elites would be less averse to allowing for nomination by primary elections. Two situations are of particular interest:

- Party elites are more moderate than the party median voter, but both are moderate. For example, $m = -b$ and $p > m$. When the conditions for a separating equilibrium

[30]If the outcome of primaries is a pooling equilibrium, then voters trade off the valence loss from the pooling equilibrium with the policy bias of party elites.

are met, adopting primaries leads to the centrist separating equilibrium, which, at least in the case of valent candidates, would deliver a policy similar to what would be chosen by party elites.

- The mirror image of the previous case is a situation in which party elites are more extreme than the median voter, but both are relatively extreme $p < m << -b$. If primaries lead to the extremist separating equilibrium, the conflict between elites and voters is partially reconciled and this might make party elites willing to allow for primaries.

Although only suggestive examples, these scenarios might also capture some elements relevant for the introduction of primaries in the United States. For example, Poole (2005) shows that party polarization was close to a historical high when primaries were introduced. This might be evidence in favor of the case of the second example mentioned above.

D General Distribution for Median Voter Location

I now relax the assumption that the median voter preferred policy is distributed uniformly in $[-b, b]$ and analyze the case of a generic distribution $f(\mu)$ which takes positive values on the same support $[-b, b]$ and has a cumulative distribution function $F(\mu)$. I assume the distribution is continuous and has no atoms. Moreover, to simplify the analysis I assume that $f(\mu)$ is symmetric around zero and that it is increasing in $[-b, 0]$ and decreasing in $[0, b]$. Finally, I assume that $\frac{f(\mu)}{F(\mu)}$ is decreasing for all μ in the support $[-b, b]$. Just like in the baseline model, I assume that the right candidate is fixed at r and has valence $v_r = v$. I also keep Assumption 1 and Assumption 2 concerning the location of primary election voters and the value of v.

Since I a am not changing preferences of voters compared to the baseline model, the indifferent voter z given two candidates still follows Lemma 1. Given an indifferent voter z, the probability for the L party candidate to win the general election is $F(z)$, which could be a non-linear function of z. Therefore, changing the distribution of the general election median voter affects the model results through $P_\theta(l) = F(z_\theta)$. In particular, in this section I will study the effects of a change in the distribution of the general election median voter on Lemma 2 and Lemma 3. Given these two building blocks, as a matter of fact, all the rest goes through practically unchanged, at least from the qualitative point of view. I start with presenting the modified version of Lemma 2.

Lemma D.1. *Suppose that the L party median voter is sufficiently to the left, in order for condition* (D.2) *to be satisfied for* $x = m$. *Then, the optimal platform chosen by a majority of primary election voters is denoted by* l^* *and satisfies condition* (D.3).

Proof of Lemma D.1

Proof. Consider a voter with $x < r$. The utility from the policy implemented by the valent candidate at r is: $-(r - x) + v$. Consider now a candidate with policy platform l and valence $v_l \in \{0, v\}$ just like in the baseline model. The utility from this candidate is $-|l - x| + v_l$. Therefore, to look for the optimal location for a candidate we are interested in the interval $[-2b - r + v - v_l, \min\{r - v + v_l, 2b - r + v - v_l\}]$. Denote the difference between the expected utility from the l candidate and the utility from the r candidate as:

$$F\left(\frac{l + r - v + v_l}{2}\right)(r - x - |x - l| + v_l - v) \tag{D.1}$$

From (D.1), it is clear that all points such that $l < x$ are dominated by $l = x$. Therefore, focus on the points in the interval $[-2b - r + v - v_l, \min\{r - v + v_l, 2b - r + v - v_l\}]$ such that $l \geq x$, so that the interval of interest becomes $[\max\{x, -2b - r + v - v_l\}, \min\{r - v + v_l, 2b - r + v - v_l\}]$. The solution l cannot be at $l = -2b - r + v - v_l$, since any point $l \leq r$ such that the l candidate wins with positive probability dominates that policy. Similarly, it cannot be at $r - v + v_l$, since any point to the left at which the l candidate wins with positive probability provides a higher utility. In order to rule out a corner solution at $2b - r + v - v_l$, which requires $r > b$, I check the first order condition of (D.1) evaluated at $2b - r + v - v_l$. This yields $f(b)(r - b + v_l - v) < 1$ and noticing that $f(b) \leq \frac{1}{2b}$, we obtain $r < 3b + v - v_l$ which is always satisfied. Therefore, the only other possible corner solution to rule out is the one at x. This requires:

$$\frac{f(z(x))}{F(z(x))} \geq \frac{2}{r - x + v_l - v} \tag{D.2}$$

Notice that the left-hand side of (D.2) is decreasing in x, whereas the right-hand side is increasing. Therefore, condition (D.2) defines an upper-bound on x such that the policy platform maximizing (D.1) is interior. If this condition is satisfied, the optimal platform satisfies:

$$\frac{f(\frac{l^* + r - v + v_l}{2})}{F(\frac{l^* + r - v + v_l}{2})} = \frac{2}{r - l^* + v_l - v} \tag{D.3}$$

□

Concerning Lemma 3, the result is an immediate consequence of the assumption on

$\frac{f(z)}{F(z)}$, as the following lemma shows.

Lemma D.2. *For any $l_2 > l_1$ and and $P_A(l_1) > 0$,*

$$\frac{P_D(l_1)}{P_D(l_2)} < \frac{P_A(l_1)}{P_A(l_2)}$$

Proof of Lemma D.2

Proof. I want to show that

$$\frac{P_\theta(l_1)}{P_\theta(l_2)} = \frac{F(z_\theta(l_1))}{F(z_\theta(l_2))} = \frac{F(\frac{l_1+r-v+v_l}{2})}{F(\frac{l_2+r-v+v_l}{2})}. \tag{D.4}$$

is increasing in $v(\theta) = v_l$. Differentiating with respect to v_l yields:

$$\frac{f(z(l_1, v_l))}{F(z(l_1, v_l))} > \frac{f(z(l_2, v_l))}{F(z(l_2, v_l))} \tag{D.5}$$

which holds for all $l_2 > l_1$ since by assumption $\frac{f(z)}{F(z)}$ is decreasing in z and $z(l, v_l)$ is increasing in l. □

With these results in place we have all it takes to construct a separating equilibrium. Therefore, there exist extremist and centrist separating equilibria that look very similar to those in the baseline model. First of all, in a separating equilibirum, non-valent pre-candidates choose policy $l_D = l^*$. Concerning valent pre-candidates, nothing changes as far as the centrist separating equilibrium is concerned. Concerning the extremist separating equilibrium, again not much changes with respect to the baseline model. The policy platform chosen by valent pre-candidates in the welfare maximizing extremist separating equilibrium is such that condition (13) is satisfied as equality for non-valent pre-candidates, which can be written as:

$$P_D(l_A^e) = aP_D(l^*) \tag{D.6}$$

Notice that given Lemma D.2, the incentive compatibility for valent pre-candidates is automatically satisfied. In order for this extremist separating equilibrium to exist, the only remaining condition is (16), stating that the valent pre-candidate at platform l_A^e is preferred to the non-valent one at l_D (in this case equal to l^*).

E Non-linear Policy Preferences

In this section, I extend the model to allow for a more general functional form to represent policy preferences. To isolate the effect of this change, I keep the distribution of the general election median voter uniform as in the baseline model. Denote the utility a voter with bliss-point x receives from the implementation of policy y as $u_x(y, v_y)$, where $y \in \{l, r\}$, $v_r = v$ and $v_l = v(\theta) \in \{v, 0\}$. I assume that $u_x(y, v_y)$ is single-peaked with bliss-point equal to x. Given two general election candidates running with platforms l and r and denoting by $\delta \equiv v_l - v_r$ the difference in valence between the two candidates, let the function $D(l, r, \delta, x) = u_x(l, v_l) - u_x(r, v)$ denote the difference in utility from candidate l and candidate r for a voter with bliss-point x. Notice that this expression allows for non-linear policy-preferences as well as for potential non-separability between valence and ideology in the utility function $u_x(y, v_y)$. I make the following assumptions on the function $D(l, r, \delta, x)$, denoting by D_j the partial derivative of $D(\cdot)$ with respect to argument j:

- $D(\cdot)$ is continuously differentiable.
- $D_l < 0$ for $l > x$ and $D_l > 0$ for $l < x$ (and $D_l = 0$ for $x = l$). Follows single-peakedness of $u_x(l, v(\theta))$.
- $D_r > 0$ for $r > x$ and $D_r < 0$ for $r < x$ (and $D_r = 0$ for $x = r$). Follows single-peakedness of $u_x(r, v(\theta))$.
- $D_{l,l} \leq 0$.
- $D_\delta > 0$
- $D_{x,l} \geq 0$
- In the case of additive separability between δ and the other variables, cross derivatives with respect to δ are null: $D_{\delta,\cdot} = 0$
- $D_x < 0$ and $\lim_{x \to -\infty} D(l, r, \delta, x) = +\infty$ and $\lim_{x \to +\infty} D(l, r, \delta, x) = -\infty$. These assumptions rule out the linear case described in the baseline model.

In this modified model, the indifferent voter, if it exists, is located at $x = z$ satisfying:

$$D(l, r, \delta, z) = 0 \tag{E.1}$$

I assume that implicit differentiation can be used without concerns on the above equation. Given z, the probability of winning the general election for a candidate with platform l is given by $\frac{z+b}{2b}$.

Lemma E.1. *Given two candidates l and r, with $l < r$, there is a unique voter indifferent between the two candidates; the indifferent voter's bliss-point is denoted by z. The following properties hold:*

- $z_l > 0$ *if* $\delta \geq 0$*, but if* $\delta < 0$*, it can be the case that* $z_l < 0$.
- $z_r > 0$ *if* $\delta \leq 0$*, but if* $\delta > 0$*, it can be the case that* $z_r < 0$.
- $z_\delta > 0$.
- $z_{l,l} \leq 0 \Leftrightarrow \frac{D_{l,l}}{D_l} \leq \frac{D_{x,l}}{D_x}$.
- $D_{l,\delta} = 0 \Rightarrow z_{\delta,l} \geq 0$.

Proof of Lemma E.1

Proof. The bliss-point of the indifferent voter z is the value of x, if it exists, solving:

$$D(l, r, \delta, x) = 0 \tag{E.2}$$

Using implicit differentiation:

$$\frac{\mathrm{d}z}{\mathrm{d}\delta} = -\frac{D_\delta}{D_x} > 0 \tag{E.3}$$

Similarly:

$$\frac{\mathrm{d}z}{\mathrm{d}l} = -\frac{D_l}{D_x} \tag{E.4}$$

The sign of (E.4) depends on D_l, which in turn depends on whether $z > l$ or $z < l$. In the former case, $\frac{\mathrm{d}z}{\mathrm{d}l} > 0$, in the latter case $\frac{\mathrm{d}z}{\mathrm{d}l} < 0$. This follows from since $D_l > 0$ for $x > l$. Concerning r,

$$\frac{\mathrm{d}z}{\mathrm{d}r} = -\frac{D_r}{D_x} \tag{E.5}$$

If $r > z$, then $D_r > 0$ and $-\frac{D_r}{D_x} > 0$. To calculate $z_{l,\delta}$, notice that:

$$\frac{\mathrm{d}^2 z}{\mathrm{d}\delta \mathrm{d}l} = -\frac{[D_{\delta,l}D_x - D_\delta D_{x,l}]}{D_x^2} = \frac{D_\delta D_{x,l}}{D_x^2} \geq 0 \tag{E.6}$$

Finally, $z_{l,l}$ is derived by:

$$\frac{\mathrm{d}^2 z}{(\mathrm{d}l)^2} = -\frac{[D_{l,l}D_x - D_l D_{x,l}]}{D_x^2} \tag{E.7}$$

from which we obtain the condition $\frac{D_{l,l}}{D_l} \leq \frac{D_{x,l}}{D_x}$ in order for (E.7) to be negative. It can be checked that this condition holds for example for the case of quadratic preferences. □

I now proceed to presenting the analogous results to Lemma 2 and Lemma 3 in this modified environment.

Lemma E.2. *The platform a voter with bliss-point x would pick for a candidate of the L party, when interior, is denoted by l^* and satisfies the following condition:*

$$-\frac{D_x(l^*)}{D(l^*)} = \frac{1}{z(l^*) + b} \tag{E.8}$$

The platform l^* is increasing in x as long as $\frac{D_{x,x}}{D_x} \leq \frac{D_x}{D}$.

Proof of Lemma E.2

Proof. The first order condition of the maximization problem of a primary election voter with bliss-point x is the following:

$$-\frac{D_l(l^*)}{D(l^*)} = \frac{z_l(l^*)}{z(l^*) + b} \tag{E.9}$$

Using the fact that $z_l(l^*) = -\frac{D_l(l^*,z)}{D_x(l^*,z)}$ and substituting this into (E.9) yields (E.8). □

Notice that in the case of quadratic policy preferences, for example, $D_{x,x} = 0$ and therefore the optimal platform l^* is increasing in x. Concerning the modified version of Lemma 3, we can show that the same result goes through, at least as long as $z(l_2) > z(l_1)$.

Lemma E.3. *For additively separable valence,*

$$\frac{P_D(l_1)}{P_D(l_2)} < \frac{P_A(l_1)}{P_A(l_2)}$$

for any $l_2 > l_1$ such that $P_A(l_1) > 0$, $P_D(l_2) > 0$, $z(l_2) > z(l_1)$.

Proof of Lemma E.3

Proof. Notice that $P_\theta(l_1) = \frac{z(l,r,\delta(\theta))+b}{2b}$. Differentiating $\frac{P_\theta(l_1)}{P_\theta(l_2)}$ with respect to δ yields:

$$\frac{z_\delta(l_1)(z(l_2) + b) - z_\delta(l_2)(z(l_1) + b)}{(z(l_2) + b)^2} \tag{E.10}$$

which is positive if and only if:

$$\frac{z_\delta(l_1)}{z(l_1) + b} > \frac{z_\delta(l_2)}{z(l_2) + b} \tag{E.11}$$

This can be rewritten as:

$$\frac{z_\delta(l_1)}{z_\delta(l_2)} > \frac{z(l_1)+b}{z(l_2)+b} \tag{E.12}$$

which can be further rearranged to:

$$\frac{D_x(l_2)}{D_x(l_1)} > \frac{z(l_1)+b}{z(l_2)+b} \tag{E.13}$$

Notice that the left-hand side is larger than one, since $D_{x,l} \geq 0$, whereas the right-hand side is lower than one as long as $z(l_2) > z(l_1)$. Therefore, as long as $z(l_2) > z(l_1)$ single crossing holds. □

Having these ingredients in place, the characterization of separating equilibria does not significantly change. Non-valent pre-candidates choose $l_D = l_m^*$, that is the policy preferred by the primary median voter. In the best separating equilibria (centrist or extremist), valent pre-candidates, instead, choose the policy satisfying the incentive compatibility constraint for non-valent pre-candidates. There are two such policies, the centrist one and the extremist one:

Lemma E.4. *In a separating equilibrium, non-valent pre-candidates choose platform $l_D = l_m^*$ defined by condition* (E.15)*. Valent pre-candidates, instead, choose the policy platform satisfying:*

$$\frac{z(l_A, r, -v)+b}{z(l_D, r, -v)+b} = a \tag{E.14}$$

where $a \equiv \frac{1-\alpha}{2-\alpha}$.

Proof. Evaluating (E.8) at $x = m$ we obtain $l_m^* = l_D$:

$$-\frac{D_x(l_m^*, m)}{D(l_m^*, m)} = \frac{1}{z(l_m^*)+b} \tag{E.15}$$

For $l_A = l_D$, clearly we have that $\frac{z(l_A,r,-v)+b}{z(l_D,r,-v)+b} = 1 > a$. For sufficiently low l_A, instead, $z(l_A, r, -v) = -b$ and so $\frac{z(l_A,r,-v)+b}{z(l_D,r,-v)+b} = 0 < a$. Given the continuity of $z(\cdot)$, therefore, a solution to (E.14) exists. In order to see that (E.14) can have at most two solutions, notice that $z_l(l, r, \delta)$ can change sign at most once, at the point $\tilde{l}$ such that $z_l(\tilde{l}, r, \delta) = \tilde{l}$, if $\delta < 0$, so there are at most two solutions, which correspond to the welfare maximizing extremist and centrist separating equilibria. □